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Third order theory based quadrilateral element for delaminated composite plates with a hybrid method for satisfying continuity at delamination fronts

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ABSTRACT

We present a four-node quadrilateral element based on the third order theory for analysis of composite plates with multiple delaminations, by employing a novel hybrid method for satisfying the continuity conditions at the delaminated fronts. In this method, the continuity of inplane displacement variables is satisfied by directly satisfying them at the midplanes of the sublaminates separated by delaminations, and employing the least squares method with respect to the shear rotation variables. The element is shown to yield very good accuracy in general, in comparison with experimental and three dimensional finite element (FE) solutions, and yield superior results to the other available analytical and FE solutions for static and free vibration responses of delaminated composite beams, and rectangular and skew composite plates under different boundary conditions. It is seen that the results from the existing continuity methods can have large error for thick beams/plates and for higher vibration modes, while the proposed hybrid method is generally very accurate and much superior to the existing methods.

1. Introduction

In laminated structures, various damage modes are possible, delamination being the most significant and severe among them. Delaminations, which are essentially debonding or separation between adjacent plies, may occur during manufacturing due to, e.g. incomplete wetting, air entrapment, large out-of-plane interlaminar stresses near free edges etc. or during service due to low velocity impact etc. The delaminations may be totally invisible or barely visible as they are embedded within the laminate, but their presence may significantly alter the vibration characteristics of the structure and/or reduce its strength, which may lead to a catastrophic failure [1,2]. The change in the vibration characteristics can also be used for its detection and structural heath monitoring purposes [3]. It is, therefore, important to model delaminations in composite structures and understand its effect on the structural response in a dynamic environment.

A review of the various analytical models and numerical analysis reported in literature for obtaining the dynamic response of delaminated composite structures has been presented by [4]. The early experimental [5] and analytical [6,7] studies on delaminated beams established the significance of effect of delaminations on the dynamic behavior of laminated structures. Most of the available methods for modelling delaminations employ the region approach, in which the sublaminates in the delaminated region are treated as individual segments connected to the intact region at the delamination fronts. Each of the sublaminates as well as the undelaminated region are modelled using the equivalent single layer (ESL) theories, which are computationally efficient because the number of displacement variables in these theories are independent of the number of layers in the laminate. Alternately, three-dimensional solid elements [8–10] or layerwise theories [11–13] have been used, in which the delaminations are modelled directly as unconnected nodes or through discontinuity functions in the displacement field, but such analyses are often too computationally involved and time consuming to be used for practical design, as the number of degrees of freedom (DOFs) increases proportionately with the number of layers in the laminate.

The earliest model on delaminated composite beams was presented by Ramkumar et al. [6], who considered a single through-width delamination and modelled the beam with four beam segments, using the first order shear deformation theory (FSDT). This model neglected the coupling effect between the axial and transverse vibrations, and the predicted natural frequencies were much

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lower than the experimental results. Later, models including the coupling effect were presented for split isotropic beams using the Euler–Bernoulli theory by Wang et al. [5] and for delaminated composite beams using the FSDT by Shen and Grady [14]. In these models, the delaminated layers are assumed to have no mutual interaction/contact between them during the deformation and they are called as the *free mode* models. The free mode models predict physically inadmissible mode shapes for off-midplane delaminations. To avoid such inadmissible mode shapes, Mujumdar and Suryanarayan [7] proposed the *constrained mode* model in which the delaminated layers are assumed to have identical transverse deformation but are allowed to slide freely relative to each other. An analytical solution was obtained using the classical beam theory. Their results and other subsequent studies [15] have shown that while the constrained mode models cannot predict the opening in the mode shapes found in experiments [14] and may yield inaccurate prediction of the fundamental frequency in such cases, when opening does not occur, their predictions do not differ much from the free mode models for the fundamental frequency.

Several FE models based on the ESL theories have been developed for dynamic analysis of delaminated beams. Andras [16] presented a FE model for composite beams with delaminations using a two-node beam element based on the classical Euler-Bernoulli beam theory in conjunction with a three-node transition element for connecting the element of the integral segment with the elements of the delaminated segment at the delamination front. The classical theory does not consider shear deformation which can be very significant in fiber reinforced polymer composite structures due to their low shear modulus to longitudinal modulus ratio. Three-node [17,18] and two-node [19–21] beam elements have been developed using the FSDT, for free vibration and wave propagation analysis of composite beams with single and multiple, full-width and partial-width delaminations, with both free mode and constrained mode assumptions. Eight-node isoparametric plate elements [22–24,17] based on the FSDT have been employed for studying the free vibration response of delaminated composite plates, based on the free mode assumption. The continuity at the delamination front is enforced by equating the displacement and rotation variables at the reference surfaces (usually midsurface) of the upper and lower elements in the delaminated region with those of the elements in the integral region at the same z-locations (called hereafter as *point continuity* method). The continuity conditions are satisfied either by the penalty parameter approach [22], which is analogous to connecting the three nodes with very stiff springs, or more exactly by applying a transformation to the element stiffness and mass matrices of the elements at the delamination front in the delaminated region [23]. Zak et al. [17] incorporated normal contact forces between the delaminated layers to prevent penetration for the forced vibration response, and Chen et al. [25] used virtual linear spring elements inserted between the delaminated sublaminates to prevent penetration between them.

FSDT is the simplest theory to account for the shear deformation effect, but it relies on arbitrary shear correction factors, whose accurate estimation particularly for dynamic problems is not trivial [26]. This motivated the development of higher order theories for the analysis of composite structures, requiring no arbitrary correction factors. Talookolaei et al. [27] have presented a three-node beam element for dynamic analysis of delaminated composite beams, using a third order theory (TOT). In the classical laminate theory (CLT) and FSDT, because the displacement field is linear across the thickness, the continuity of displacement and rotation variables at the reference surface ensures the continuity of the displacements at all points across the sublaminates. However, in case of TOT, due to the nonlinear variation of the displacement field, the aforementioned continuity conditions which are usually adopted [27–29] do not satisfy the continuity across the entire thickness at the delamination front, nor is it trivial to achieve the same.

The TOT of the type of Reddy [30] requires C^1-continuity of the deflection variable, which is difficult to be achieved directly in a quadrilateral element. A four-node rectangular element based on the TOT has been used by Radu and Chattopadhyay [28] to analyze the dynamic stability of composite plates with delaminations, considering the free mode model. The same element was employed by Kumar and Shrivastava [29] to study the free vibration response of square laminated plates having a central cutout with delamination around the cutout. They employed the point continuity conditions at the midsurfaces of the sublaminates. Park et al. [31] studied the free vibration analysis for composite skew plates with delaminations around quadrilateral cutouts using a four-node element based on the TOT. They employed the point continuity at the bottom surface for the lower sublaminate and at the top surface for the upper sublaminate. Subsequently, Noh and Lee [32] have investigated the dynamic instability of delaminated skew plates, using the same element. Hu et al. [15] developed an FE model based on the Reddy's TOT for moderately thick composite laminates containing delaminations by using a nine-node C^0-continuous isoparametric element having seven DOFs per node, and adopting the penalty function method for relating the rotational DOFs to the derivatives of the deflection. In an attempt to satisfy the continuity condition at the delamination fronts more accurately, instead of doing it at a single point, the continuity of the inplane displacements was sought to be satisfied across the sublaminates using the least square method. However, no comparison of this method with the point continuity method was reported.

Kulkarni and Kapuria [33] developed a four-node quadrilateral element for stress analysis of laminated composite plates, based on the TOT, in which the C^1-continuity requirement was circumvented by using the discrete Kirchhoff (DKQ) technique, originally proposed by Batoz and Tahar [34] for thin plate bending elements. The element was shown to be much superior to the other available FSDT and TOT based elements in terms of accuracy, and free from shear locking as well.

In this work, we extend the four-node quadrilateral element of Kulkarni and Kapuria [33] based on the TOT for the dynamic analysis of composite plates with multiple delaminations, by employing a novel method for satisfying the continuity conditions at the delaminated fronts. In this method, the continuity of inplane displacement variables is sought to be satisfied across the thickness of the sublaminates separated by delaminations by directly satisfying them at their midplanes, and employing the least square method with respect to the shear rotation variables. The inadequacies of both point continuity and least square continuity methods for thick delaminated plates are illustrated for the first time, and the performance of proposed hybrid point-least square continuity method is illustrated in comparison with the 3D elasticity based solutions. The results are presented for both static and free vibration responses.

2. Third order theory approximations

Consider a multilayered composite plate having multiple delaminations both across the laminate thickness and along the spans, as shown in Fig. 1(a). Using the region approach, the delaminated regions are divided into a number of sublaminates separated by the delaminations, and each of these sublaminates is modeled separately. In the present formulation, the displacement field in each sublaminate/intact laminate is assumed to follow the TOT approximations, which ensure the shear traction-free conditions at the top and bottom surfaces of the sublaminate/laminate. Fig. 1(b) shows a sublaminate of thickness h with its