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Analysis of mechanical properties of laminated rubber bearings based on transfer matrix method

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ABSTRACT

Laminated rubber bearings are constructed from alternating layers of rubber vulcanized to reinforcing steel shims, therefore, they can be treated as periodic structures. The transfer matrix method (TMM) is developed to analyze mechanical behaviors of the periodic bearings subjected to a compressive axial load and a lateral shear deformation. The periodic model can account for the effect of thicknesses, material properties and boundary conditions of each rubber and steel layer, on the details of mechanical behaviors between interlaminated layers. Haringx's theory for the individual rubber layer and rigid body motion theory for the steel shim are utilized in the development of the model. Hence, the proposed approach overcomes Haringx's theoretical drawback, which assumes the entire laminated rubber bearings as an equivalent continuous column, and is also a robust alternative to the stiffness matrix method through a complex assembly of a number of identical periodic elements. Comparisons with Haringx's results show that TMM has good accuracy for analyzing laminated rubber bearings. The introduction of the pattern of periodic structure along with TMM provides a viable means for determining mechanical properties of laminated rubber bearings with different configurations by only changing the number of periodic elements and thicknesses of the layers.

1. Introduction

Seismic isolation using multilayer elastomeric bearings has become a widely accepted technique to reduce the seismic load transmitted to the structure. An elastomeric laminated rubber bearing consists of periodic alternating layers of thin rubber reinforced with steel shims, thus providing high vertical stiffness to carry the weight of a structure and yet maintaining horizontal flexibility to shift the fundamental natural frequency of the isolated structure away from the dominated frequencies of strong ground motion [1–3]. Thus, to estimate the behavior of an isolated structure, it is necessary to develop an accurate model to study the mechanical properties of the rubber isolated bearings.

The theoretical approaches usually adopted by researchers to study the mechanical properties of the laminated rubber bearings were based on Haringx's theory [4,5], which assumed the bearing as an equivalent continuous column taking into account the influence of axial compressive load and shear deformation. Ravari et al. [6], for example, have applied the theory to investigate the mechanical properties of multilayer elastomeric bearings by considering the initial end rotations. The Haringx's type formulation, for predicting the horizontal stiffness of elastomeric bearings, is observed to be consistent with experimental results for moderate amounts of shear strains [7,8]. One of the most popular problems involving the application of Haringx's theory is the stability of multilayer rubber bearing, traced back to Gent [9] who studied the stability of multilayer rubber compression springs. The buckling load and buckling behavior of laminated elastomeric bearings were fully investigated by Stanton et al. [10] and Imbimbo and Kelly [11,12]. The axial load effects in such theories can also be explained by a simple two-spring model [13,14], which can reflect the mechanical property and stability problem of the laminated rubber bearing. Kikuchi et al. [15] then developed a new analytical model comprising of multiple shear springs and axial springs, by extending to three dimensions an existing spring model for the elastomeric bearing. Further nonlinear analytical models based on the two-spring model [13,14] were proposed in Refs. [16–18] to predict the force-displacement behavior of elastomeric bearings. The effectiveness of these nonlinear models was evaluated and verified using results from experimental study performed by Buckle et al. [19] and Sanchez et al. [20]. However, the above

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theoretic investigations of the laminated rubber bearings based on the model of Haringx's equivalent column cannot address the details of the mechanical behaviors between interlaminated rubber layers and cannot handle different thicknesses, material properties, cross section areas and boundary conditions layer by layer [21]. To solve these problems, Chang [21] has proposed the element stiffness matrix method to study the mechanical behavior of laminated rubber bearings. In their approach, a closed form stiffness matrix of a single rubber layer was derived, through complicated solutions of four different degrees of freedom, and a stiffness summation procedure for all layers was also needed to obtain a global stiffness matrix and equilibrium equation.

Another approach, the transfer matrix method (TMM), was also used to investigate the behavior for laminated rubber bearings. Mazda and Shiojiri [22] were pioneering in the application of TMM to the simplified analysis of laminated elastomers. Then, Takayama et al. [23] also used the method to evaluate the mechanical characteristic of laminated rubber bearings. It is promising that TMM is adopted for analyzing the displacement of laminated elastomers which has the advantages of simplicity and fewer iterations so as to meet the boundary conditions. Also, for periodic structures, Ruzzene and Baz [24] utilized the method of the transfer matrix to investigate the characteristics of wave propagation in periodic shells. In addition, Yeh and Chen [25] predicted the pass and stop bands for a periodic sandwich beam by the finite element method based on the transfer matrix.

In the present study, a periodic structure model along with TMM, because of its simplicity, is developed to solve the mechanical properties of laminated rubber bearings. The model is formulated according to Haringx's theory for the rubber layer, while the steel shims are considered as rigid body. The state vector at any intermediate layer can be determined using the transfer matrix. Different from the approach presented in Ref. [22], the proposed method employs directly the general forms of the solutions for two differential equations of moment and shear equilibrium instead of introducing the linear rate of rotation change in order to obtain the transfer matrix of each element in the rubber layer. Furthermore, the proposed approach overcomes the drawbacks of Haringx's theory, and is also used as an alternative to the stiffness matrix method because it avoids a complex assembly of a number of identical periodic elements. The accuracy of the developed model is validated by Haringx's theory using laminated rubber bearings with different configurations.

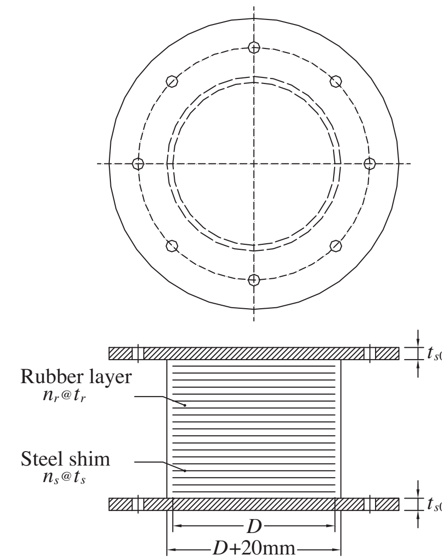

Fig. 1. Typical laminated rubber bearing.

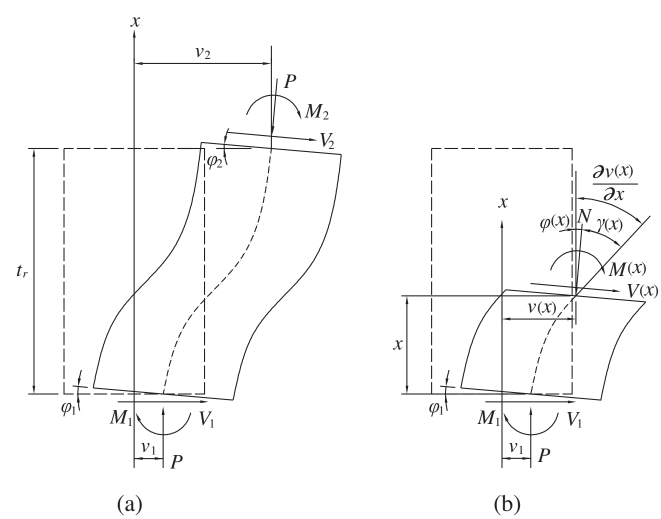

Fig. 2. Geometry and loads of rubber column: (a) full configuration; (b) middle section.

2. Formulation of laminated rubber bearing

2.1. Governing equation of a rubber layer

The typical laminated rubber bearing, as shown in Fig. 1, consists of alternating layers of rubber vulcanized to reinforcing steel shims. The basic periodic element in the bearing includes one rubber layer bonded to one steel shim. The rubber layer is modeled as a continuous column in which plane sections normal to the undeformed axis remain plane but not necessarily normal to the deformed axis, and the deformation of one rubber layer of height t_r in a periodic element j is shown in Fig. 2(a). The x-axis denotes the centroidal axis of the rubber column with the origin located at the bottom of the undeformed column. The deformation is described by two parameters $v(x)$ and $\varphi(x)$, which are lateral displacement of the centroidal axis and the rotation of a section normal to the undeformed axis, respectively, as shown in Fig. 2(b). The internal forces on the deformed plane section are the shear force $V(x)$ parallel to the section and the bending moment $M(x)$. The constitutive equations for bending moment and shear force can be expressed in terms of the deformation quantities as

$$M(x) = EI\frac{\partial\varphi(x)}{\partial x} \tag{1a}$$

$$V(x) = GA\left[\frac{\partial v(x)}{\partial x} - \varphi(x)\right] \tag{1b}$$

where EI, G, A are effective bending stiffness, shear modulus and the total shear area, respectively.

When the rotation $\varphi(x)$ is small, using Eqs. (1a) and (1b), the equations of moment and shear equilibrium in the deformed state shown in Fig. 2(b), are

$$EI\frac{\partial\varphi(x)}{\partial x} + P\,[v(x) - v_1] + M_1 - V_1 x = 0 \tag{2a}$$

$$GA\left[\frac{\partial v(x)}{\partial x} - \varphi(x)\right] - P\,\varphi(x) + V_1 = 0 \tag{2b}$$

where the axial load P, the horizontal load V_1 and the moment M_1 are the external loads applied to the bottom ($x = 0$) of the rubber layer; v_1 is the lateral displacement at $x = 0$.