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Research Paper

# Computation of the safety factor for slope stability using discontinuous deformation analysis and the vector sum method

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ABSTRACT

This study introduces the vector sum method into discontinuum-based methods by considering the sliding vector and the stress state of the discrete block system. The sliding direction computation and force projection in the new approach are detailed, and the safety factor is solved by explicit equations. The vector sum method is implemented in the discontinuous deformation analysis (DDA) program and is used to compute the safety factors for two numerical examples. A comparison of the solutions obtained with the theoretical analysis and limit equilibrium analysis demonstrates that the new method is suitable for calculating the safety factor of a slope.



## 1. Introduction

There are two common calculation methods in slope stability analysis: the limit equilibrium method (LEM) [1] and the finite element method (FEM) [2]. Compared to the FEM, the LEM is simple and has high computational efficiency, which have made it the fundamental method for slope stability analysis [3]. However, the LEM is a static analysis method and ignores the movement of the slope under the action of external forces. Large displacements, discontinuous contacts, a precise friction law and stabilized time-step dynamic computation are required to reproduce the slope movement until the equilibrium state. Discontinuous deformation analysis (DDA) is a new efficient numerical analysis method that can meet the above requirements [4].

Since it was first proposed by Shi [5], the DDA method has been developed rapidly in many respects, such as contact and 3D analysis [6–14]. DDA can accurately simulate the translation, rotation and deformation of a block based on time-step calculations. Therefore, it is able to not only analyze the sliding instability but also simulate slope failure, such as toppling and collapse. Yeung [15] verified the form of sliding and toppling failure of blocks. MacLaughlin et al. [16] simulated both the plane and circular failure modes of a slope and found that the results have better accuracy than the analytical solutions or the results of conventional analytical methods. Cheng et al. [17] analyzed the accuracy of DDA in simulating the four basic motion modes of a rock, namely, free-falling, rolling, sliding and bouncing. The DDA method has also been used to analyze the stability of practical slopes. Hatzor et al. [18] investigated the dynamic deformation of the upper terrace of King Herod's Palace in Masada. Jiao et al. [19] and Xu et al. [20] simulated the entire processes for three practical landslides, namely, the Qianjiangping, Majiagou and Dongmiaojia landslides. Wu et al. [21,22] applied DDA to simulate the kinematic behavior of sliding rock blocks in the Tsaoling and Chiu-fen-erh-shan landslides, which were induced by the 1999 Chi-Chi earthquake. Zhang et al. [23] and Wu et al. [24] presented a run-out analysis of the Daguangbao landslide subjected to near-fault multi-direction earthquake forces. Huang et al. [25] used DDA to model the Donghekou landslide, which was triggered by the 2008 Wenchuan earthquake.

In addition to being used to simulate the failure process of a slope, DDA can be used to compute the safety factor. There are three ways to define the safety factor in DDA. The first approach is the fictitious force method [26], where the safety factor is defined as the magnification of the fictitious force, which puts the slope in a state of critical instability. The second approach is the strength reduction method [27], where the safety factor is defined as the ratio of the reduced shear strength parameters, which puts the slope in a state of critical instability. The third

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approach is the contact force method [28], where the safety factor is defined as the ratio of the resistant and sliding forces, which are calculated with the contact forces of the sliding surface. In the former two approaches, the stress states are obtained by multiple calculations under hypothetical conditions, and the safety factor cannot truly represent the actual state of the slope. The third approach is actually the LEM, and it considers the dynamic iterative process using DDA. However, in this approach, integrations of the sliding or resistant forces are the scalar sum of the contact force components, which cannot reflect the concept of the sliding vector. To overcome this problem, Ge et al. [29] proposed the vector sum method (VSM), in which the safety factor is computed based on the real stress state and the vector sum algorithm, and thus, the stress field only needs to be calculated once and the mechanical meaning is clear [30,31].

In this paper, the VSM has been introduced into DDA to determine the safety factor, and two key steps—sliding direction computation and force projection—are detailed considering the dynamic iterations and contact forces.

## 2. A brief description of DDA

A discrete deformable block is the basic unit of the DDA method, and the individual blocks are connected and form the block system based on the boundary conditions. DDA uses time-step calculations. For each time step, based on the simplex integration method, the stiffness, inertia, initial stresses, loading, and contact matrices are collected to form the following total equilibrium equations:

$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} \Delta D_1 \\ \Delta D_2 \\ \vdots \\ \Delta D_n \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} \quad (1)$$

where $[K_{ij}]$ is a stiffness sub-matrix of order $6 \times 6$, $\{F_i\}$ is a load sub-vector of order $6 \times 1$, and $\{\Delta D_i\}$ is the incremental displacement solution of a block in a time step: $i, j = 1, 2, \ldots, n$, where $n$ is the block number.

The most important task for solving the total equilibrium equations is accurately calculating the contact forces between the blocks. In the DDA method, three contact modes are defined: open, lock and slide. These contact modes are converted in the calculation process by the addition or subtraction of normal or tangential springs at each contact position. We call this approach "open-close" iterations, which should satisfy two principles: no penetration or tension in the normal direction and Coulomb's law in the tangential direction.

In the two-dimensional DDA method, all contacts between the blocks are treated in the angle-edge form. As shown in Fig. 1, P1 is a vertex of block i, and $P_2P_3$ is an edge of block j. In the current time step, before solving the total equilibrium equations, the motion parameters in the previous time step are used to estimate the movements, and $P_1$ is assumed to move to $P_0$, which may be located in block j; then, and the penetrations are generated. $d_N$ and $d_S$ are the normal and tangential components of the penetrations, respectively, and the normal and tangential components of the hypothetical contact forces are expressed as

$$\left.\begin{aligned} R_n' &= K_n d_N \\ R_s' &= K_s d_S \end{aligned}\right\} \quad (2)$$

where $R_n'$ and $R_s'$ are the normal and tangential components of the hypothetical contact forces, respectively, and $K_n$ and $K_s$ are the normal and tangential spring stiffness coefficients, respectively.

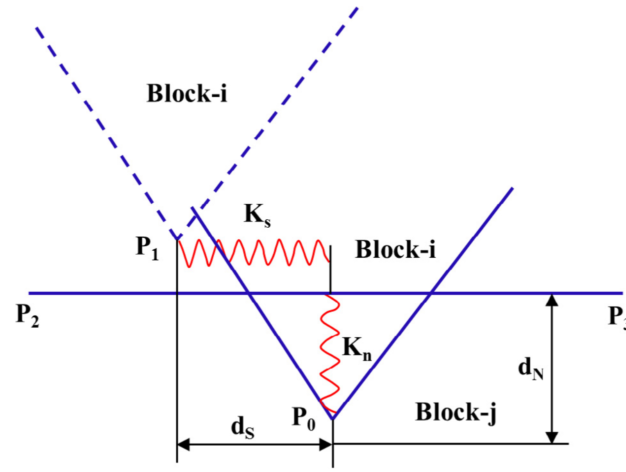

**Fig. 1.** Angle-edge contact in DDA.

Then, the total equilibrium equations are solved, and the motion parameters in the current time step are obtained. The contact modes and penetrations are updated, and the hypothetical contact forces are modified to obtain the true contact forces. $R_n$ and $R_s$ are the normal and tangential components of the true contact forces, respectively, and they are confirmed by the following conditions:

(a) If $R_n'$ is positive (it is assumed that tension is positive and pressure is negative),

$$R_n' = K_n d_N \geqslant 0 \quad (3)$$

In this case, the contact mode is open, and no normal or tangential springs are added. The true contact forces are expressed as

$$\left.\begin{aligned} R_n &= 0 \\ R_s &= 0 \end{aligned}\right\} \quad (4)$$

(b) If $R_n'$ is negative and $R_s'$ is less than the shear strength,

$$\left.\begin{aligned} R_n' &= K_n d_N < 0 \\ R_s' &< R_n' \tan\varphi + cl \end{aligned}\right\} \quad (5)$$

where $\varphi$ and $c$ are the friction angle and cohesion strength, respectively, of the contact surface and $l$ is the contact length.

In this case, the contact mode is lock, and both the normal and tangential springs are added. The true contact forces are equal to the hypothetical contact forces, which are expressed as

$$\left.\begin{aligned} R_n &= R_n' \\ R_s &= R_s' \end{aligned}\right\} \quad (6)$$

(c) If $R_n'$ is negative and $R_s'$ is not less than the shear strength,

$$\left.\begin{aligned} R_n' &= K_n d_N < 0 \\ R_s' &\geqslant R_n' \tan\varphi + cl \end{aligned}\right\} \quad (7)$$

In this case, the contact mode is slide, and only the normal spring is added. The true contact forces are expressed as

$$\left.\begin{aligned} R_n &= R_n' \\ R_s &= R_n' \tan\varphi \end{aligned}\right\} \quad (8)$$

In the original DDA program written by Shi [5], if slide has occurred, the cohesive force is reduced to zero and the friction angle remains unchanged in the next time-step calculation, which reflects the sliding friction of the contact surface. A literature review shows that the errors are generally lower than 1% if the