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Research Paper

Prediction of fracture trajectory in anisotropic rocks using modified maximum tangential stress criterion

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ABSTRACT

A new criterion to predict crack propagation trajectory in anisotropic rocks with incorporating the concept of *T*-stress in formulating stress field near the crack tip was developed. The developed criterion along with enrichment functions and interaction integral in the extended finite element method (XFEM) framework made a sophisticated tool in modeling fracturing process in anisotropic media. Numerical results indicated that stress intensity factors considerably depend on orientation of anisotropy axes and ratio of the elastic modulus. The proposed formulation for anisotropic media provides a more accurate prediction of crack propagation trajectory compared with conventional methods, especially in mixed mode conditions.

1. Introduction

There are many deficits such as cracks, joints and fractures in rock structures. Consequently, when a rock is subjected to mechanical loading considering different environmental factors, it probably fails and new cracks possibly extend from the tips of preexisting discontinuities. Evaluating the stress intensity factor and predicting the crack propagation trajectory are highly important in different fields of rock engineering works such as hydraulic fracturing, underground excavation, rock mass stability analysis, hydrocarbon reservoirs, and blasting operations [1–6]. Furthermore, predicting the crack propagation path provides valuable information on optimizing stone blocks (in building-stone quarries), the stability of rock structures and their post-destruction volume (in rock slopes), and the efficient analysis of hydraulic fracturing in hydrocarbon and geothermal reservoirs. In reality, cracked structures and rock masses are often subjected to complex loading conditions, and their failures mostly occur due to the simultaneous contribution of several loads. Thus, the fracture of cracked components and structures may grow along non-straight paths, and not necessarily in the direction of the initial crack [7]. Therefore, investigating the crack initiation angle and the crack propagation path under mixed loading mode are the favorite subjects for researchers in the field of rock mechanics. Several theoretical models [8–13] and experimental techniques [14–18] have been developed to investigate the mixed mode crack growth (the combination of opening and shearing modes) in rocks. However, the theoretical models are limited to simple geometries, loads and uncomplicated behavior of materials. An anisotropic rock has different elastic moduli, strengths, Poisson's ratios and permeability properties in different directions. The anisotropic case is much more complex than the isotropic case. Many previous studies are based on the isotropic and continuous assumption, simply for necessity and/or convenience of obtaining closed form solutions [19]. In practice, however, engineers must deal with discontinuous anisotropic rocks. One method to overcome these weaknesses is to use numerical modeling techniques. Today, numerical methods are used as an efficient tool for problem-solving in complex conditions. One such efficient numerical method used to model crack initiation and propagation is the Extended Finite Element Method (XFEM). Its basic concept is to enrich the local solution by applying a partition of unity framework to a standard Finite Element Method [20]. The fracturing process is modeled on the basis of the enrichment of polynomial approximation space, adding new functions to the approximation space, and thereby increasing the degrees of freedom of nodes. For example, by employing the discontinuous Heaviside function, crack surfaces can be modeled without considering them as geometric boundaries. In addition, the singularity of stress

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field near a crack tip can be reproduced by applying the asymptotic displacement functions. Sih et al. [21] and Viola et al. [22] used the notion of complex numbers to introduce analytical solutions for calculating stress fields near a crack tip in anisotropic materials, and these were further developed by other researchers [20,23,24]. The greatest advantage of the XFEM approach is mesh-independency, while maintaining all the important advantages of the finite element method, such as solving nonlinear problems and ductile behavior of materials. Extensive studies using this method have been conducted within a short time since its introduction, indicating that this method has a unique capability in modeling problems associated with fracture mechanics [25–29].

There are several failure criteria in predicting the direction of crack initiation and propagation trajectory such as the Maximum Tangential Stress (MTS) [8], minimum strain energy density [12], and maximum energy release rate [9], which have been frequently used by researchers in the field of rocks and geo-materials [30–32]. The MTS criterion in which the crack is propagated from the crack tip along the direction of the maximum tangential stress has been used more extensively in XFEM for modeling crack growth [33]. However, deviations of the MTS outputs from experimental results have encouraged many researchers to find an alternative or improve this method [34–38]. Bobet [36] compared the results of crack initiation angle by the MTS criterion with the results obtained from uniaxial compressive tests, and stated that this criterion did not offer a suitable estimation of the crack initiation angle. He also presented a stress-based criterion in order to estimate the crack initiation angle by using the Boundary Element Method (BEM). Shen and Stephansson [37] also developed the "*F*" criterion or the modified "*G*" criterion for the same purpose. Mohtarami et al. [39,40] used such an approach to introduce an improved MTS theory based on the approximation of stress field near the crack tip in XFEM. Smith et al. [41] utilized the *T*-stress as well as the singular terms of the crack tip to develop a criterion called the GMTS to predict the crack initiation and the propagation trajectory. They stated that the mixed mode toughness of a cracked specimen depended on the magnitude and sign of *T*-stress; a notion that had been ignored in previous studies. Aliha and Ayatollahi tested this criterion for a variety of rock specimens and confirmed its validity [42–44]. They reported that this criterion was well capable to predict the crack trajectory. It should to be noted that the asymmetric distribution of pores, joints, discontinuities, and different mineral compositions in rocks cause them to exhibit an anisotropic behavior [19]. In addition, according to Mohtarami et al. [40], some engineering operations such as acidizing oil wells can also intensify the anisotropic behavior of surrounding rocks. Despite the merits of extensive studies conducted in this field, all of the above-mentioned criteria have been developed for isotropic media, while they may not be adapted for anisotropic materials [20,23,45]. Anisotropy in the mechanical properties of rocks could control the stability of underground and open excavations and foundations in civil engineering, drilling, blasting, and rock cutting in mining engineering, and borehole deviation, stability, deformation and failure in petroleum engineering. Thus, the proper incorporation of anisotropy into computer simulations is necessary for finding the accurate solution to various engineering problems. The main objective of this research work is to develop a criterion to predict the fracture trajectory of anisotropic rocks by XFEM simulation. In this case, the concept of *T*-stress initially developed by Smith et al. [41] is incorporated into the stress field near the crack tip in an anisotropic medium. Then, with the help of *T*-stress and anisotropic crack-tip enrichment functions, the Anisotropic Maximum Tangential Stress (AMTS) criterion is developed to predict the crack initiation angle and the fracture trajectory in such a media.

Following the concept of *T*-stress, the method to calculate the stress and the displacement field near the crack tip in an anisotropic body as developed by Sih et al. [21] is described. This method is then utilized to develop the MTS criterion for anisotropic solids. In Section 3, the XFEM approach, enrichment functions and interaction integral in anisotropic bodies are described to reproduce near-crack tip stresses and calculate the stress intensity factors. According to the literature, due to the availability and simple preparation of core-based specimens in estimating rock fracture toughness, the Cracked Brazilian Disc (CBD) [46–48], Hollow Centre Cracked Disc (HCCD) [49–51], Flattened Brazilian Disc [52,53], Modified Ring [54,55], and Semi-circular Bend (SCB) [56–58] specimens are commonly used. Thus, the validity and accuracy of the proposed method for predicting the crack propagation trajectory are investigated for three types of isotropic and anisotropic disc-shaped cracked specimens, and the results are compared with the obtained experimental test results and the results reported in the literature.

2. AMTS criterion for mixed mode fracture in an anisotropic media

The linear elastic plane stress near the crack tip can be described as symmetric and asymmetric fields called Mode *I* and Mode *II*, respectively. The stresses of each field can be expressed as series expansion of eigenvalues [59]; with general form as follows,

$$\sigma_{ij} = \underbrace{\begin{array}{l}\textit{singular terms of}\\ \textit{stress around crack tip}\end{array}}_{\text{function of } K_I, K_{II}, r, \theta} + \underbrace{T-stress}_{\begin{array}{l}\textit{appears only in}\\ \textit{Mode I and for } \sigma_{xx}\end{array}} + \underbrace{O(r^{1/2})}_{\text{Higher order terms}} \quad (i,j = x,y) \tag{1}$$

where x and y are Cartesian coordinates, and r and θ are polar coordinates defined with respect to a crack tip. Parameters K_I, and K_{II} are stress intensity factors in mode *I* and *II*, and T is the *T*-stress, which all depend on the geometry and the configuration of loading, and they can vary significantly in different specimens [41]. Near the crack tip, higher order terms of the series are negligible [60]. In the conventional MTS criterion [8], only the singular term of Eq. (1) is considered, but for the criterion introduced in our developed model, the impacts of both the singular term and the *T*-stress are considered. Thus, the two terms of right side in Eq. (1) yield the stress fields near the crack tip, and can be used in predicting the crack propagation trajectory. A constant stress parallel to the crack, term T, appears only due to the symmetrical component of loading, and it will be zero in pure mode *II* [61]. Several methods have been developed to calculate T for different loading conditions and geometries [62–65]. According to the literature, the provided formulation must give T at a reasonable distance from the crack tip. But in practice, the finite element results are not acceptable, unless a large number of elements is used in the simulation of the crack tip zone [66,67]. The enrichment functions, however, allow the stress singularity and the discontinuity-induced displacements to reproduce accurately without any fine mesh generation or re-meshing process. In this context, Ayatollahi et al. [61] developed an improved method to obtain *T*-stress by using the displacements along crack surfaces without any dependency on the number of mesh elements. They stated that for a mixed mode *I*/*II*, term T can be expressed as follows,

$$T = \frac{1}{2}E'\left(\left(\frac{du_x}{dx}\right)_{\theta=-\pi} + \left(\frac{du_x}{dx}\right)_{\theta=\pi}\right). \tag{2}$$

where u_x represents the displacement along the x-axis, and E' is defined as follows,

$$E' = \begin{cases} E & \text{planes tress} \\ \frac{E}{1-\nu^2} & \text{plane strain} \end{cases} \tag{3}$$