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Research Paper

Critical-state-based Mohr-Coulomb plasticity model for sands

Sang Inn Woo [a,*], Hoyoung Seo [b], Joonyoung Kim [a]

[a] Department of Civil and Environmental Engineering, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, South Korea
[b] Department of Civil, Environmental, and Construction Engineering, Texas Tech University, 911 Boston Ave., Lubbock, TX, USA

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ABSTRACT

This paper presents a refined Mohr-Coulomb model for sands based on the critical state theory. The refined model adjusts a dilatancy angle based on the state parameter with respect to the critical state line. Furthermore, a friction angle is decomposed into the critical state friction angle and a portion of the dilatancy angle to capture the peak phenomenon of dilative sands. The elemental simulations of the drained and undrained triaxial compression tests on Toyoura sand using the refined model showed much better performance than the conventional Mohr-Coulomb model.

1. Introduction

Sands are particulate materials whose mechanical properties are primarily dependent on the confinement and initial density. Despite their complex behavior, the Mohr-Coulomb (MC) plasticity model has been widely used to describe their mechanical constitutive relationship (or stress-strain relationship). In the field of geotechnical engineering, the MC model has been used for simulations of shallow foundations [1–5], pile foundations [6], cone penetration tests [7], offshore anchors [8], soil slopes [9], and deep penetrations [10–18]. To define its yield surface, the MC model relies on two strength parameters: friction angle ϕ and cohesion c. For cohesionless materials ($c = 0$ and $\phi > 0$) such as sands, the MC model has an anisotropic conical yield surface with a tip at the origin of the principal stress space. The associated flow rule in the MC model that assumes dilatancy angle ϕ_d is identical to friction angle ϕ overestimates the dilatancy (plastic volumetric deformation caused by plastic shear deformation). Therefore, to avoid such overestimation, the non-associated flow rule ($\phi_d \neq \phi$) has been adopted to realistically describe the dilatancy phenomenon in numerical simulations of sands [2,6,7,18].

According to Bolton [19], the friction angle ϕ of sands can be decomposed into

$$\phi = \phi_{cs} + 0.8\phi_d \tag{1}$$

where ϕ_{cs} is the critical state friction angle, which depends on the intrinsic properties of sand (e.g., mineralogy of sand). Bolton [19] stated that dilatancy angle ϕ_d goes to zero and friction angle ϕ becomes identical to critical state friction angle ϕ_{cs} after prolonged shearing. This implies that dilatancy angle ϕ_d and hence friction angle ϕ of sand change during shearing. For simplicity, however, friction angle ϕ and dilatancy angle ϕ_d have been typically kept constant during shearing in many numerical studies on sands [2,6,7,18,20]. The objective of this study is to refine the MC model to better capture the dilatancy of sands with the help of the critical state soil mechanics. In the refined MC model proposed in this study, dilatancy angle ϕ_d is defined as a function of the state parameter proposed by Been and Jefferies [21] with respect to the critical state; the dilatancy angle continuously evolves during shearing and eventually reaches zero at the critical state in this model.

The formulation of the proposed refined MC plasticity model is presented, including the summary of the critical state theory, in Section 2, followed by the calibration and performance evaluation of the proposed model for Toyoura sand in Section 3. Section 4 then concludes the paper. The proposed model was formulated based on effective stress σ'_{ij} ($=\sigma_{ij} - u\delta_{ij}$, where u is the pore water pressure), and the geomechanics sign convention where compression takes a positive sign was used in this study.

* Corresponding author.
E-mail addresses: sanginnwoo@gmail.com (S.I. Woo), hoyoung.seo@ttu.edu (H. Seo), goldenrain91@gmail.com (J. Kim).

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2. Constitutive model formulation

2.1. Yield surface

The present study relies on the rounded Mohr-Coulomb (MC) yield criterion [22,23]. For sand (or cohesionless soil), it can be written as

$$f = \sqrt{J_2}K - p' \sin\phi = 0 \tag{2}$$

where ϕ is the friction angle, p' is the mean effective stress (= $\sigma'_{pp}/3$), J_2 is the second invariant of a deviatoric stress (= $s_{ij}s_{ij}/2$ where deviatoric stress $s_{ij} = \sigma'_{ij} - p'\delta_{ij}$) and K is a function that makes the corners of the yield function smooth. Lode's angle θ is defined by

$$\theta = \frac{1}{3}\sin^{-1}\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right) \tag{3}$$

where J_3 is the third invariant (or determinant) of a deviatoric stress tensor (= det(**s**) = $s_{ij}s_{jk}s_{ki}/3$). Equation (3) implies that $\theta = 30°$ under triaxial compression loading conditions whereas $\theta = -30°$ under triaxial extension loading conditions. When the magnitude of the current Lode's angle θ is less than the limit Lode's angle θ_T, the function K is defined by

$$K = \cos\theta - \frac{1}{\sqrt{3}}\sin\phi\sin\theta \quad \text{if}|\theta| < |\theta_T| \tag{4}$$

which makes the MC yield function (Equation (2)) identical to the classical MC yield function. If the magnitude of θ is greater than θ_T (i.e., if the current stress is near the corners of the MC yield surface), the function K is defined by

$$K = A - B\sin 3\theta \quad \text{if}|\theta| \geqslant |\theta_T| \tag{5}$$

where A and B are functions of Lode's angle θ, limit Lode's angle θ_T, and friction angle ϕ and are given as

$$A = \left[\cos\theta_T + \tfrac{1}{3}\cos\theta_T\tan\theta_T\tan 3\theta_T\right] + \left[\tfrac{1}{3\sqrt{3}}cos\theta_T(tan3\theta_T - 3tan\theta_T)\right]\text{sign}(\theta)sin\phi \tag{6}$$

$$B = \frac{\sqrt{3}}{9}\frac{\cos\theta_T}{\cos 3\theta_T}\sin\phi + \frac{1}{3}\frac{\sin\theta_T}{\cos 3\theta_T}\text{sign}(\theta) \tag{7}$$

The combination of equations (5)–(7) provides smooth (differentiable) corners of the MC yield surface. Fig. 1 shows the traditional MC yield surface (thick gray line) and the rounded MC yield surface (thin black line) together in the π plane, where friction angle $\phi = 35°$ and limit Lode's angle $\theta_T = 25°$. Fig. 1 clearly shows the smooth corners of the rounded MC yield surface, where the magnitude of Lode's angle $|\theta|$ is greater than that of limit Lode's angle $|\theta_T|$.

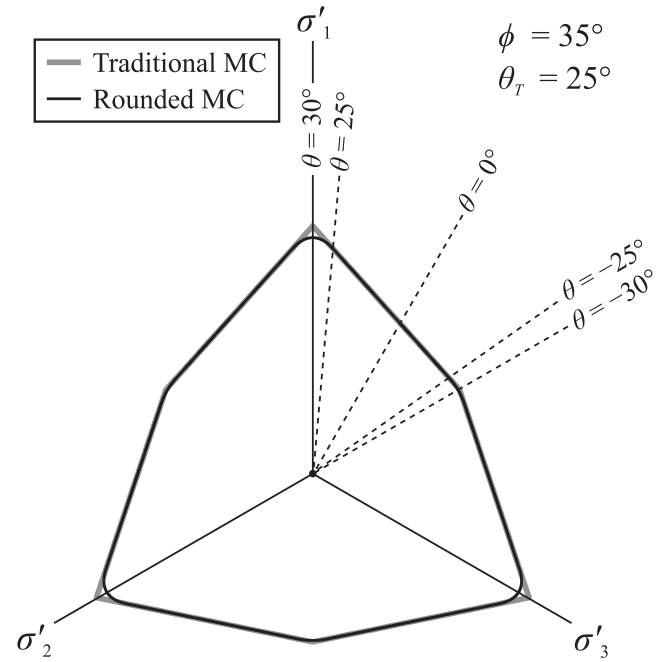

Fig. 1. Traditional and rounded Mohr-Coulomb yield surfaces in the π plane.

2.2. Critical State (Steady State)

The critical state is the state that a saturated soil eventually reaches upon prolonged shearing [24]. According to Li and Dafalias [25], the mathematical expression of the critical state is

$$\frac{\partial p'}{\partial t} = 0, \ \frac{\partial q}{\partial t} = 0, \ \frac{\partial e}{\partial t} = 0, \ \frac{\partial \varepsilon_q}{\partial t} \neq 0 \tag{8}$$

where t is the time, q (= $(3J_2)^{1/2}$) is the representation of the octahedral shear stress, e is the void ratio of sand, and ε_q (= $(2/3e_{ij}\ e_{ij})^{1/2}$ where $e_{ij} = \varepsilon_{ij} - \varepsilon_{pp}\ \delta_{ij}\ /3$) represents the octahedral shear strain. Equation (8) implies that void ratio e, shear stress q, and mean effective stress p' are constants at the critical state; therefore, the critical state is typically expressed based on e, q, and p' in a mathematical modeling of soils. Fig. 2 shows the locus of the critical state in the p'-q-e space (Fig. 2(a)) and in the p'-e plane (Fig. 2(b)) accompanied by drained and undrained paths upon shearing. In Fig. 2(a), the stress paths in the p'-q-e space are represented as thick lines whereas those projected on the p'-q and p'-e planes are shown as thin lines. Regardless of the drainage condition and an initial void ratio (hollow circle), the p'-q-e state of sand converges to the critical state (solid circle) after prolonged shearing. The projection of the locus of the critical state onto the p'-e plane is called the critical state line (denoted as CSL in Fig. 2(a) and (b)) whereas its projection onto the p'-q plane (or stress space, more generally) is named the critical state surface (denoted as CSS in Fig. 2(a)). Based on the experiment data obtained from Verdugo and Ishihara [26], Li [27] and Li and Wang [28] proposed an equation for the CSL of sands, as follows:

$$e_{cs} = \Gamma_{cs} - \lambda\left(\frac{p'}{p_a}\right)^{\xi} \tag{9}$$

where e_{cs} is the void ratio at the critical state, p_a is the reference pressure (= 98.1 kPa), Γ_{cs} is the e_{cs} at $p' = 0$, and λ and ξ are the material constants that take positive values. To represent how far the current e-p' state is from the critical state, Been and Jefferies [21] defined state parameter ψ_{cs} as the difference between current void ratio e and void ratio e_{cs} at the critical state at the same mean effective stress p' (refer to Fig. 2(b)).

$$\psi_{cs} = e - e_{cs} \tag{10}$$

As shown in Fig. 2(b), state parameter ψ_{cs} geometrically represents the vertical distance to the critical state line in the e-p' space, and takes a positive or negative value depending on whether current void ratio e is located above or below the CSL. Upon prolonged shearing, the loading path of the sands in the e-p' space eventually reaches the CSL, and state parameter ψ_{cs} becomes zero at the critical state.

The critical state surface, which is the relation between q and p' at the critical state, is typically expressed through the critical state stress ratio M_{cs} (= $(q/p')_{cs}$). Critical-state friction angle ϕ_{cs} can be written as a function of the critical state stress ratio $M_{cs,TXC}$ under triaxial compression conditions by