Computers and Geotechnics 92 (2017) 210–219

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

A simple method to describe three-dimensional anisotropic failure of soils

Yu Tian, Yang-Ping Yao *

School of Transportation Science and Engineering, Beihang University, Beijing 100191, China

ARTICLE INFO

Article history:
Received 7 March 2017
Received in revised form 9 July 2017
Accepted 8 August 2017

Keywords:
Anisotropy
Failure criteria
Fabric
Modified stress
Soils

ABSTRACT

In order to describe the anisotropic failure of soils caused by the internal fabric, isotropic failure criterion should be generalized to be anisotropic. This paper achieves the generalization by introducing a simple method, called anisotropic transformed stress method, which apparently differs from the common way. Physical interpretation of this method are analyzed further. Using this method, many existing isotropic criteria can become anisotropic in the same way, and will be expressed by a unified formula finally. To verify this method, anisotropic Unified Strength Criterion is used to predict the peak strength of anisotropic soils in different loading conditions.

1. Introduction

Numerous experimental studies show that the peak strength of soil varies considerably with the loading direction [1,2]. This is usually attributed to the preferred oriented internal structure in soils [3,4]. In order to describe the anisotropic failure of soils, Wang [5] proposed a general mathematical expression:

$$f(\boldsymbol{\sigma},\mathbf{F}) = f(\mathrm{tr}\boldsymbol{\sigma},\mathrm{tr}\boldsymbol{\sigma}^2,\mathrm{tr}\boldsymbol{\sigma}^3,\mathrm{tr}\mathbf{F},\mathrm{tr}\mathbf{F}^2,\mathrm{tr}\mathbf{F}^3,\mathrm{tr}\boldsymbol{\sigma}\mathbf{F},\mathrm{tr}\boldsymbol{\sigma}^2\mathbf{F},\mathrm{tr}\boldsymbol{\sigma}\mathbf{F}^2,\mathrm{tr}\boldsymbol{\sigma}^2\mathbf{F}^2) = 0 \tag{1}$$

where σ_{ij} is the stress tensor and F_{ij} is the fabric tensor which is a measure of the microstructure.

For practical applications, an efficient way to develop anisotropic failure criteria is to generalize the existing isotropic criteria, such as the extended Mises criterion, Mohr-Coulomb criterion, Lade-Duncan criterion [6] and Matsuoka-Nakai criterion [7], to be anisotropic. To develop anisotropic Mohr-Coulomb criterion, Pietruszczak and Mroz [8] assumed that the friction angle and cohesion were functions of the microstructure tensor and loading vector. Then they found a critical plane on which the shear-normal stress ratio reached the peak strength first. A polynomial of higher order was employed by Azami et al. [9] to modify the friction angle and cohesion, so that the numerical approximation became more accurate. Schweiger et al. [10] and Gutierrez et al. [11] established an interpolation function based on friction angles in the vertical and horizontal directions. Guo and Stolle [12] derived the relation between the friction angle and internal fabric by considering micromechanics. Based on the Lade-Duncan criterion, Lade [13] developed a three-dimensional anisotropic failure criterion by changing the right hand of the equal sign to be an orientation-dependent variable. Same idea was adopted to extend the Matsuoka-Nakai criterion by Xiao et al. [15] and Kong et al. [15]. Besides, Mortara [16] used a shape function of Lode's angle to revise the failure locus in the deviatoric plane. Gao and Zhao [17] introduced a joint invariant of the stress tensor and fabric tensor into the Generalized Nonlinear Strength criterion [18], so that the strength anisotropy of different geomaterials, such as clays, sands and rocks, could be characterized with satisfaction. Liu and Indraratna [19] used a coefficient, which was related to the angle between the bedding plane and the Spatial Mobilized Plane (SMP), to discount the stress ratio in their general isotropic failure criterion for geomaterials. Another stress-based variable, defined as the "Euclidean distance" between the bedding plane and the maximum shear stress ratio plane, was adopted by Cao et al. [20]. From the above methods, we can find some similarities: first, a scalar-valued variable, which essentially represents the loading direction with respect to the material direction, is derived from the stress tensor and/or fabric tensor; then strength parameters in the isotropic criteria are made functions of this variable. These methods can fit well with test results. In the review article of Duveau et al. [21], the anisotropic failure criteria developed by

* Corresponding author at: School of Transportation Science and Engineering, Beihang University, 37 Xueyuan Road, Haidian District, Beijing, China.
E-mail address: ypyao@buaa.edu.cn (Y.-P. Yao).

http://dx.doi.org/10.1016/j.compgeo.2017.08.004
0266-352X/

these methods were classified into "empirical continuous models". Duveau et al. [21] also pointed out that a high number of tests were needed to obtain reliable empirical laws for the modification of parameters.

In this paper, instead of strength parameters, the stress tensor in the isotropic criteria is modified based on the anisotropic transformed stress (ATS) method [22]. This method was originally proposed to establish a constitutive model called anisotropic Unified Hardening (UH) model [22], and was able to account for the non-coaxiality of soils [23] automatically. After illustrating its physical meaning in terms of describing strength anisotropy, we find that this method is versatile to generalize many existing isotropic failure criteria to be anisotropic. A simple and unified formula, which can degrade into different criteria, can be obtained in the anisotropic transformed stress space. One of the anisotropic criteria developed by this method is used to predict the peak strength of anisotropic soils in different loading conditions, and the comparisons once again confirm the validity of the ATS method.

2. Anisotropic transformed stress method

As an inherent property of soils, anisotropy can significantly influence the peak strength. In order to describe this, anisotropic failure criterion should be established. Based on the existing isotropic failure criterion, anisotropic criterion can be conveniently developed by introducing a quantity that measures the directionality of material properties. There are two problems along this line: (1) what is a proper quantity to describe soil anisotropy, and (2) how to introduce this quantity into the isotropic criterion. The following will explain how these two problems are solved in this paper.

2.1. Fabric tensor

Fabric tensor measures the spatial distribution of the orientation of particles, voids or contact normals in soils [24,25]. According to some microscopic studies [26–28], there exist some qualitative or even quantitative relations between microstructural and macromechanical properties of geomaterials. For example, because soil particles tend to lie down with long axes being parallel to the bedding plane, shear sliding is hard to occur when the soil is loaded from the vertical direction. As a result, the vertical strength is usually higher. That is to say, as a measure of the microstructure, fabric tensor is also capable of reflecting the strength anisotropy. So fabric tensor will be used as the anisotropic quantity to develop anisotropic failure criteria. For most sedimentary soils, the fabric is cross-anisotropic, i.e., the distribution of particles is symmetric around one axis (denoted by $\mathbf{e}'_1$), while in the bedding plane (denoted by $\mathbf{e}'_2 - \mathbf{e}'_3$) the distribution is homogeneous. Therefore, in the space of $(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3)$, fabric tensor can be expressed by principal values as follows

$$F'_{ij} = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \frac{1}{2}(1-\Delta) & 0 \\ 0 & 0 & \frac{1}{2}(1-\Delta) \end{pmatrix} \tag{2}$$

where Δ measures the concentration degree of long axes towards the direction of $\mathbf{e}'_1$. $\Delta = 1/3$ means the material is isotropic (in this case, $F'_{ij} = \delta_{ij}/3$, where δ_{ij} is the Kronecker delta). For sedimentary soils, Δ is usually smaller than 1/3. If the space $(\mathbf{e}'_1, \mathbf{e}'_2, \mathbf{e}'_3)$ does not coincide with the reference frame $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, orthogonal transformation must be done as follows

$$F_{ij} = \beta_{ik}\beta_{jl}F'_{kl} \tag{3}$$

where $\beta_{ij}\ (= \mathbf{e}_i \cdot \mathbf{e}'_j)$ is an orthogonal tensor. Fabric tensor works as a measure of strength anisotropy in the anisotropic failure criteria hereinafter, and a practical approach to determine its value will be adopted if no microstructural statistical data is available.

2.2. Modified stress tensor and its physical interpretation

It is very cumbersome to introduce a tensor-valued quantity into the failure criteria. In the references, some researchers derived a scalar-valued variable from the stress tensor and fabric tensor [8,9,13–15,17], while others introduced the fabric tensor into the stress invariants [29–31]. Unlike them, this paper adopts a modified stress tensor, which is a tensor-valued function of σ_{ij} and F_{ij}, to establish failure criteria directly. The modified stress tensor is defined as

$$\bar{\sigma}_{ij} = \frac{3}{2}(\sigma_{ik}F_{kj} + F_{ik}\sigma_{kj}) \tag{4}$$

If the material is isotropic ($F_{ij} = \delta_{ij}/3$), the modified stress tensor $\bar{\sigma}_{ij}$ reduces to the ordinary stress tensor σ_{ij}. Note that for simplicity, only the product of σ_{ij} and F_{ij} is introduced to define the modified stress tensor, which will render the failure criteria unable to reflect the dependence of peak strength on direct invariants of σ_{ij} and F_{ij}. However, it will be proved later that this simplified treatment does not bring large error in the prediction. Besides, we should declare that the idea of the modified stress tensor was first proposed by Tobita and Yanagisawa [32], but they used the inverse of F_{ij}, rather than direct F_{ij}, in their expression. Here we will provide the physical interpretation of the stress modification and explain why such an expression of $\bar{\sigma}_{ij}$ is used.

The stress modification aims to create an equivalence between the anisotropic soil with a 'virtual' isotropic soil. To illustrate this, the stress mapping from σ_{ij} to $\bar{\sigma}_{ij}$ in a simple loading condition is projected onto the deviatoric plane, as shown in Fig. 1. In this condition, principal directions of both σ_{ij} and F_{ij} are coaxial with the coordinate axes. Point V and Point H represent the failure stress state of triaxial compression in which the major principal stress is along the vertical direction and horizontal direction, respectively. Because the vertical peak strength is usually higher than the horizontal, q_v (deviatoric stress of Point V) is larger than q_h (deviatoric stress of Point H). Therefore, it is impossible to draw an isotropic failure locus which passes through Points V and H simultaneously. However, according to the stress modification (Eq. (4)), a modified stress tensor can be obtained as follows

$$\bar{\sigma}_{ij} = \begin{pmatrix} 3\Delta \cdot \sigma_Z & 0 & 0 \\ 0 & \frac{3}{2}(1-\Delta)\cdot\sigma_X & 0 \\ 0 & 0 & \frac{3}{2}(1-\Delta)\cdot\sigma_Y \end{pmatrix} = \begin{pmatrix} \bar{\sigma}_Z & 0 & 0 \\ 0 & \bar{\sigma}_X & 0 \\ 0 & 0 & \bar{\sigma}_Y \end{pmatrix} \tag{5}$$

where σ_Z, σ_X and σ_Y are principal values of σ_{ij}; $\bar{\sigma}_Z$, $\bar{\sigma}_X$ and $\bar{\sigma}_Y$ are principal values of $\bar{\sigma}_{ij}$. Considering $\Delta < 1/3$, one has $\sigma_Z > \bar{\sigma}_Z$, $\sigma_X < \bar{\sigma}_X$ and $\sigma_Y < \bar{\sigma}_Y$. Therefore, after the stress modification, Point V moves downward to Point $\bar{\text{V}}$ in the modified stress space, while Point H moves to Point $\bar{\text{H}}$, as shown in Fig. 1. Observe that Point $\bar{\text{H}}$ is not located in the $\bar{\sigma}_Y$ -axis. This is because in this loading condition (see the bottom right of Fig. 1), the two minor principal stresses, σ_Z and σ_X, are modified differently due to the anisotropic fabric along the Z- and X-axes. And hence, one will get $\bar{\sigma}_Z < \bar{\sigma}_X < \bar{\sigma}_Y$ so that the modified stress state is true triaxial. The deviation of Point $\bar{\text{H}}$ from the $\bar{\sigma}_Y$-axis just reflects the effect of anisotropy. It can be seen from Fig. 1 that the gap between the vertical and horizontal peak strengths is narrowed after the stress modification, so that we can connect Points $\bar{\text{V}}$ and $\bar{\text{H}}$ by an isotropic failure locus. That is to say, in the modified stress space, the anisotropic soil can be treated to be isotropic, and its strength can be modeled by isotropic