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Research Paper

Efficient system reliability analysis of rock slopes based on Subset simulation

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ABSTRACT

How to efficiently assess the system reliability of rock slopes is still challenging. This is because when the probability of failure is low, a large number of deterministic slope stability analyses are required. Based on Subset simulation, this paper proposes an efficient approach for the system reliability analysis of rock slopes. The correlations among multiple potential failure modes are properly accounted for with the aid of the "max" and "min" functions. A benchmark rock slope and a real engineered rock slope with multiple correlated failure modes are used to demonstrate the effectiveness of the proposed approach.

1. Introduction

Slope instability can be catastrophic events often leading to loss of life and property. It is widely recognized that there are multiple failure modes induced by structural planes in rock masses (e.g., [16,8,38,35,27]). How to assess the system reliability of rock slopes have received extensive attentions recently. For example, Low [30–32] investigated the system reliability of a rock wedge with four failure modes by employing the First Order Reliability Method (FORM). Jimenez-Rodriguez et al. [21] and Jimenez-Rodriguez and Sitar [22] used a disjoint cut-set formulation in which each cut set corresponds to a failure mode. Li et al. [25,26] proposed a n-dimensional equivalent method to study the system reliability of rock slopes with multiple correlated failure modes. Lee et al. [24] developed a knowledge-based clustered partitioning (KCP) technique for the system reliability analysis of rock wedges. Johari and Lari [23] performed system reliability analysis of rock wedges with four correlated failure modes using a sequential compounding method (SCM). It is noted however, how to effectively evaluate the system reliability of rock slopes at low probability levels has not been investigated substantially. Although direct Monte-Carlo simulation (MCS) has the advantage of robustness and conceptual simplicity (e.g., [3,39]), it is computationally intensive when the probability of failure is small. Therefore, it is necessary to develop more efficient methods for the system reliability analysis of rock slopes at low probability levels.

Moreover, there often exist multiple failure modes in rock slopes due to pre-existing discontinuities. Although the correlations between the failure modes can be easily taken into account if numerical simulation techniques such as finite element methods and direct MCS are used together (e.g., [17]), it is usually too time consuming. It is thus preferable to use analytical performance function approaches to replace direct numerical simulations. It is noted however, that the correlations among different potential failure modes should be properly addressed (e.g., [30,25]). As reported in Li et al. [26], it led to a significant overestimation of the system failure probability of the rock slope using the Cornell's bound method [11] and disjoint cut-set formulation [21] since the correlation between pairs of potential failure modes is ignored. Several approaches have been developed to model the correlations among different failure modes, for example, the Ditlevsen's bound method [13], probabilistic fault tree model (e.g., [41,25,26]), first order approximation method [21] and SCM [23]. A biased estimate of probability of system failure could be obtained because equivalent linear performance functions were used to compute

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the correlation coefficients. Thus, it is of significance to effectively and properly model the correlations among the multiple potential failure modes of rock slopes.

This study proposes an efficient approach for the system reliability analysis of rock slopes based on Subset simulation. The effectiveness of the proposed approach is demonstrated through a benchmark rock slope and the Jinping I left abutment slope in China. The expressions for the estimating of system probabilities of failure for parallel, series and combined slope systems using simulation techniques are derived. The "max" and "min" functions are utilized to construct the system performance functions. Then, the basic theory of Subset simulation is explained and the implementation of the proposed approach is presented. Finally, reliability analysis of two rock slope examples is carried out to show the effectiveness of the proposed approach.

2. System reliability of rock slopes

From a reliability point of view, systems can be classified as parallel, series and parallel-series systems [3]. This classification is also applicable in the reliability analysis of rock slopes. The probability of slope failure, $p_{f,s}$, is computed by the following integral over the system failure domain [9]:

$$p_{f,s} = P[G_{sys}(\boldsymbol{X}) < b] = \int_{G_{sys}(X)<b} f_X(X)dX \tag{1}$$

where $P(.)$ represents the probability of an event; $G_{sys}(.)$ denotes the performance function of slope system; b is the threshold value which usually equals 0 for the slope failure event; $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)^{\mathrm{T}}$ is the vector of random variables, in which n is the number of random variables; $f_{\boldsymbol{X}}(\boldsymbol{X})$ is the joint probability density function of $\boldsymbol{X}$.

It is virtually impossible to evaluate the n-fold integral directly for slope stability problems because the domain of integration is very complicated. However, simulation techniques such as direct MCS can be employed to evaluate this integral (e.g., [3,44,19]),

$$p_{f,s} = P[G_{sys}(\boldsymbol{X}) < b] = \frac{1}{N_{sim}}\sum_{k=1}^{N_{sim}} I[G_{sys}(\boldsymbol{X}_k) < b] \tag{2}$$

where N_{sim} is the number of simulations; $I[.]$ is an indicator function. For a given random sample $\boldsymbol{X}_k$, $k = 1, 2, \ldots, N_{sim}$, $I[G_{sys}(\boldsymbol{X}_k) < b]$ is set to 1.0 when $G_{sys}(\boldsymbol{X}_k) < b$. Otherwise, it is set to zero. The performance function $G_{sys}(.)$ can either be analytical or numerical. If numerical methods such as finite element method are adopted to assess the system performance, the calculated probability of failure is for the whole system. Generally, the numerical methods are computationally more expensive than the analytical performance function approaches. In this study, the "max" and "min" functions are adopted to construct the system performance functions (e.g., [4,14,7,18]). For a parallel system, the system will only fail if all components fail. In other words, the probability of system failure is smaller than that of its any components. The probability of failure for a parallel system consists of N_p components can be expressed as

$$\begin{aligned} p_{f,s} &= P[G_{sys}(\boldsymbol{X}) < b] = P\left(\cap_{i=1}^{N_p} E_i\right) \\ &= \frac{1}{N_{sim}}\sum_{k=1}^{N_{sim}} I\left[\max_{i=1,2,\cdots,N_p} g_i(\boldsymbol{X}_k) < b\right] \end{aligned} \tag{3}$$

where E_i stands for the failure event along the ith failure mode. For a series system, the reliability of the system requires that none of its components fail. In other words, the failure of any of its components will lead to the failure of the entire system. The probability of failure of a series system that is made up of N_s components can be expressed as

$$\begin{aligned} p_{f,s} &= P[G_{sys}(\boldsymbol{X}) < b] = P\left(\cup_{i=1}^{N_s} E_i\right) \\ &= \frac{1}{N_{sim}}\sum_{k=1}^{N_{sim}} I\left[\min_{i=1,2,\cdots,N_s} g_i(\boldsymbol{X}_k) < b\right] \end{aligned} \tag{4}$$

The probability of failure of a combined system with N_s sub-parallel systems, each of which consists of N_p components, can be defined as

$$\begin{aligned} p_{f,s} &= P[G_{sys}(\boldsymbol{X}) < b] = P\left(\cup_{j=1}^{N_s}\cap_{i=1}^{N_p} E_{i,j}\right) \\ &= \frac{1}{N_{sim}}\sum_{k=1}^{N_{sim}} I\left\{\min_{j=1,2,\cdots,N_s}\left[\max_{i=1,2,\cdots,N_p} g_{i,j}(\boldsymbol{X}_k)\right] < b\right\} \end{aligned} \tag{5}$$

Similarly, the probability of failure of another type of combined system with N_p sub-series systems, each of which consists of N_s components, can be estimated as

$$\begin{aligned} p_{f,s} &= P[G_{sys}(\boldsymbol{X}) < b] = P\left(\cap_{j=1}^{N_p}\cup_{i=1}^{N_s} E_{i,j}\right) \\ &= \frac{1}{N_{sim}}\sum_{k=1}^{N_{sim}} I\left\{\max_{j=1,2,\cdots,N_p}\left[\min_{i=1,2,\cdots,N_s} g_{i,j}(\boldsymbol{X}_k)\right] < b\right\} \end{aligned} \tag{6}$$

The probability of system failure can be generally estimated by the direct MCS once the system performance function is formulated. This is feasible when the probability of failure is relatively high. However, the direct MCS will suffer from a serious lack of efficiency for the cases with small probabilities of failure (e.g., $p_{f,s} < 10^{-5}$). Thus, to calculate such small probabilities of system failure of rock slopes involving multiple correlated failure modes, Subset simulation is utilized to reduce computational costs.

3. Subset simulation-based system reliability analysis approach

Subset simulation originally proposed by Au and Beck [5] is used to estimate the small probability of failure as a product of relatively large conditional probabilities of some intermediate failure events. Subset simulation has been widely applied to effectively analyze high-dimensional component reliability problems such as slope reliability (e.g., [6,42,37,28,29,20]), but the system reliability problems have been rarely investigated with Subset simulation. To our best knowledge, only Bourinet et al. [10], Zio [46], Au and Wang [7] and Papaioannou et al. [34] studied structural system reliability problems based on Subset simulation. With the system performance functions constructed using the "max" and "min" functions in Section 2, Subset simulation is extended in this section to tackle the system reliability problems of rock slopes at low probability levels.

3.1. Subset simulation

The probability of slope system failure, $p_{f,s}$, can be expressed as

$$p_{f,s} = P(F) = P[G_{sys}(\boldsymbol{X}) < b] = P(F_1)\prod_{i=2}^{m} P(F_i|F_{i-1}) \tag{7}$$

where F stands for the event of slope system failure; m is the number of levels required to reach the system limit state surface; $F_i = \{G_{sys}(\boldsymbol{X}) < b_i\}$, $i = 1, 2, \ldots, m$, are a set of the intermediate system failure events that are defined by a decreasing sequence of intermediate threshold values $b_1 > b_2 > \ldots > b_{m-1} > b_m = 0$, respectively; $P(F_1) = P[G_{sys}(\boldsymbol{X}) < b_1]$ is the probability of system failure corresponding to the first level of Subset simulation; $P(F_i|F_{i-1}) = P[G_{sys}(\boldsymbol{X}) < b_i|G_{sys}(\boldsymbol{X}) < b_{i-1}]$ is the intermediate conditional probability of system failure. It is not a trivial task to determine b_i in advance. Alternatively, the values of b_i are chosen adaptively so that the estimated probabilities of $P(F_1)$ and $P(F_i|F_{i-1})$, $i = 2, 3, \ldots, m-1$, are equal to a fixed value p_0 that is referred to as conditional failure probability. A commonly specified value of $p_0 = 0.1$ is used in this study.