Computers and Geotechnics 82 (2017) 67–84

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Method of deformation analysis for composite structures of soils and masonry stones

Ryota Hashimoto [a], Mamoru Kikumoto [b], Tomofumi Koyama [c,*], Mamoru Mimrua [a]

[a] Department of Urban Management, Kyoto University, Japan
[b] Institute of Urban Innovation, Yokohama National University, Japan
[c] Faculty of Societal Safety Sciences, Kansai University, Japan

ARTICLE INFO

Article history:
Received 20 June 2016
Received in revised form 31 August 2016
Accepted 26 September 2016

Keywords:
NMM-DDA
Elasto-plastic constitutive law
Node-based uniform strain element
Deformation analysis
Soil and masonry composite structure
Bearing capacity

ABSTRACT

The coupled numerical manifold method (NMM) and discontinuous deformation analysis (DDA) are enhanced to simulate deformations of continuous soil and discontinuous masonry structures. An elasto-plastic NMM-DDA is formulated that incorporates elasto-plastic constitutive laws into incremental forms of the equation of motion. A node-based uniform strain element is applied to avoid volumetric locking, which often occurs in conventional NMM-DDA. The proposed method is applied to three fundamental boundary value problems: a beam bending problem, a bearing capacity problem of a footing, and a bearing capacity problem of a masonry structure. The method is verified through comparisons with conventional solutions.

1. Introduction

Worldwide, there are many historic monuments that were constructed with man-made soil foundations and stones, and some of them are in danger of collapsing because, over time, their weight has induced deformation of the ground beneath them. For instance, many monuments in the Angkor ruins of Cambodia [1], which usually have the soil foundation shown in Fig. 1(a), have suffered damage and finally collapsed because of the uneven settlement of their foundations, as shown in Fig. 1(b). To restore such historic structures requires a remedy that can achieve structural stability and simultaneously conserve the original construction method, and the various possible restoration methods should therefore be evaluated and compared with regard to these objectives before work begins. From a geotechnical perspective, evaluating the effects of the various restoration techniques requires a rational method of evaluating the stability of the masonry structures by precisely simulating the collapse mechanisms. Because a masonry structure's stability problem is governed by the mechanical interaction between the masonry stones and the ground, the analysis method must incorporate both the discrete bodies of the masonry stones and the continuum bodies of the soils.

Many numerical methods have been developed to address discontinuity in the context of continuum analysis. The most popular of these is the finite element method (FEM) with the zero-thickness interface elements [2–4]. This technique enables the modeling of the discontinuous behavior between the continuum bodies. However, the application of the method to the behavior of masonry structures is quite limited because many interfacial surfaces exist between stones and between the stones and soil, and it is difficult to model perfect separation, rotation, and generation of new contact surfaces for several reasons, such as node connectivity.

In order to overcome these limitations, several novel approaches have been developed and applied. One of the popular methods that incorporates both continua and discontinua is a method combining FEM and the discrete element method (DEM) [5]. Munjiza [6] proposed a combined FEM-DEM approach that discretizes each discrete element (block) by finite elements and considers contacts between discrete blocks by DEM; this method was applied to the analysis of dry stone masonry structures [7,8].

Abbreviations: CST, constant strain triangular [element]; DDA, discontinuous deformation analysis; ME, manifold element; NB, node-based uniform strain [element]; NMM, numerical manifold method.

* Corresponding author.
E-mail addresses: hashimoto.ryouta.57u@st.kyoto-u.ac.jp (R. Hashimoto), kikumoto@ynu.ac.jp (M. Kikumoto), t-koyama@kansai-u.ac.jp (T. Koyama), mimura.mamoru.3r@st.kyoto-u.ac.jp (M. Mimrua).

http://dx.doi.org/10.1016/j.compgeo.2016.09.011

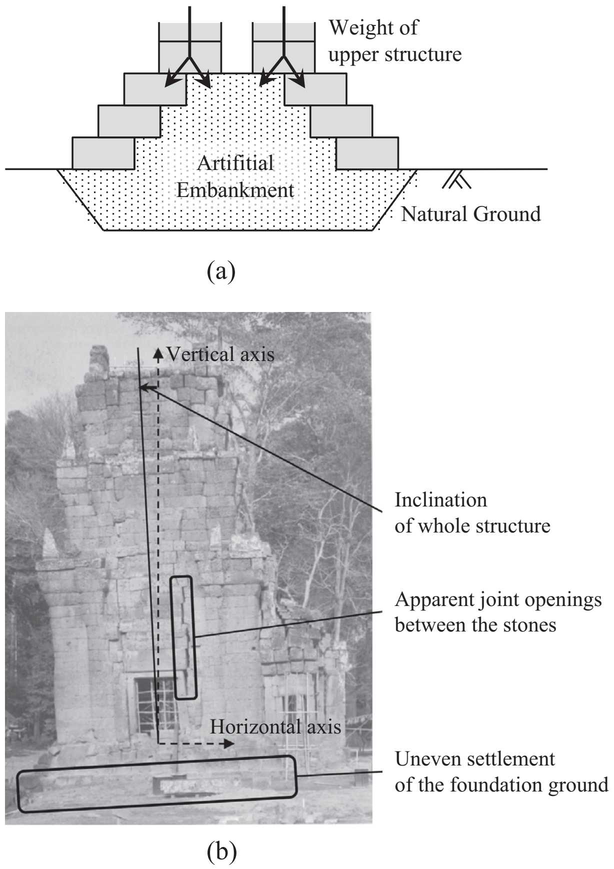

Fig. 1. Damaged masonry structure of Angkor ruin, Cambodia: (a) schematic figure of masonry platform and (b) Prasat Suor Prat N1 Tower [1].

Michael et al. [9] developed another combined FEM-DEM scheme that incorporates the contacts between the finite elements and the particle discrete elements, and applied this method to a tire-terrain interaction problem. However, the applicability of the FEM-DEM approach to quasi-static and long-term problems is rather limited because it treats contacts based on DEM algorithms that explicitly integrate the equation of motion. Because this scheme requires significantly small time steps to satisfy the governing equation, it is very difficult for the combined FEM-DEM method to obtain an accurate solution that satisfies the equilibrium of the whole soil and masonry system.

On the other hand, Miki et al. [10] proposed coupling a discontinuous deformation analysis (DDA) [11] with the numerical manifold method (NMM) [12], which are numerical methods for the dynamic and quasi-static analyses of contact between discontinuous elastic bodies. This coupled NMM-DDA method achieves strong coupling of the behavior of continua and discontinua. The difference between the two methods is the discretization of the displacement field in the materials. DDA employs rigid body displacement, rotation, and strain at the centroid of the block as the unknown variables, and can properly simulate rockfall [13], dynamic landslide of a rock slope [14,15] and dynamic stability analyses of the masonry stones [16–19]. In contrast, NMM takes the nodal displacements of a mesh composed of finite mathematical covers over the analytical domain as the unknown variables, and can consider the detailed deformation of the continuum as well as the contacts. In the coupled NMM-DDA, an analytical domain consisting of multiple discrete continua is divided into those continua modeled by DDA and those modeled by NMM, and the simultaneous analysis is achieved by solving the deformation of both domains and the contacts among them. Because the original DDA and NMM utilize the same contact treatment algorithms and implicit time integration of the equation of motion, the strong coupling of whole analytical domain consisting of both continua and discontinua can be properly achieved.

In the present study, a new numerical method for solving the interaction problem of the soil and the masonry structure is developed by extending the NMM-DDA, and the applicability of the proposed method is discussed through several numerical examples. In the proposed method, the discontinuous behaviors of the masonry stones are modeled by DDA, and the deformation of the soil is described by NMM, newly extended with the elasto-plastic constitutive model. This enables a more realistic description of the mechanical interactions between the masonry structures and the ground. However, volumetric locking [20] results in an unrealistically stiff solution when a nonlinear stress-strain relationship is used as an elasto-plastic model for the soils and implemented in the conventional NMM-DDA, which employs constant strain triangular elements for the NMM domain, as shearing with the constant volume at the critical state cannot be captured. Hence, a method is applied to avoid volumetric locking in NMM.

This paper first provides the governing equations of the contact problem of continua and their weak forms in Section 2, and the formulation of the elasto-plastic NMM-DDA is shown in Section 3. A method to avoid volumetric locking in NMM using a node-based uniform strain element [21] is explained in Section 4. In Section 5, several numerical examples are presented and solved, such as the elastic cantilever bending problem typically affected by volumetric locking, and the bearing capacity problems of the strip footing under central or eccentric loading. Finally, in Section 6, the NMM-DDA is applied to the bearing capacity problem of the masonry platform consisting of soil and the masonry structure.

2. Governing equations of continuum kinematics and mutual contact problem

NMM-DDA analyzes the mechanical behaviors of systems that consist of multiple continua by considering their mutual contacts. This problem is governed by the equation of motion for each continuum and additional constraint conditions that must be satisfied wherever contacts occur among the objects. A system that includes n independent continua is hereafter assumed. The domains occupied by each continuum are named as Ω_i $(i = 1, 2, \cdots, n)$, and the boundary surfaces of Ω_i are named as Γ_i. The kinematic problem of a single material Ω_i, shown in Fig. 2, is described with the equation of motion:

$$\rho_i \ddot{\boldsymbol{u}}_i - \nabla \cdot \boldsymbol{\sigma}_i - \bar{\boldsymbol{b}}_i = 0 \quad \text{in } \Omega_i; \tag{1}$$

the strain compatibility condition:

$$\boldsymbol{\varepsilon}_i = \frac{1}{2}\{\nabla \boldsymbol{u}_i + (\nabla \boldsymbol{u}_i)^{\mathrm{T}}\}; \tag{2}$$

and the incremental form of the constitutive equation:

$$\Delta \boldsymbol{\sigma}_i = \boldsymbol{D}_i : \Delta \boldsymbol{\varepsilon}_i. \tag{3}$$

In Eqs. (1)–(3), the subscript i indicates the physical quantity of Ω_i, the dot $(\cdot)$ means the material derivative, ρ_i is the density, $\boldsymbol{u}_i$ is the displacement vector, $\boldsymbol{\sigma}_i$ is the Cauchy stress tensor, $\bar{\boldsymbol{b}}_i$ is the known body force vector, $\boldsymbol{\varepsilon}_i$ is the infinitesimal strain tensor, and $\boldsymbol{D}_i$ is the constitutive relations tensor. In addition, the displacement field must satisfy the displacement boundary conditions:

$$\boldsymbol{u}_i = \bar{\boldsymbol{u}}_i \quad \text{on } \Gamma_{iu}; \tag{4}$$

and the stress boundary conditions:

$$\boldsymbol{t}_i = \boldsymbol{\sigma}_i \cdot \boldsymbol{n}_i = \bar{\boldsymbol{t}}_i \quad \text{on } \Gamma_{i\sigma}. \tag{5}$$