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Research Paper

An edge-based smoothed point interpolation method for elasto-plastic coupled hydro-mechanical analysis of saturated porous media

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A B S T R A C T

An edge-based smoothed point interpolation method is adopted for coupled hydro-mechanical analysis of saturated porous media with elasto-plastic behaviour. A novel approach for the evaluation of the coupling matrix of the porous media is adopted. Stress integration is performed using the substepping method, and the modified Newton-Raphson approach is utilised to address the nonlinearities arising from the elasto-plastic constitutive model used in the formulation. Numerical examples are studied and the results are compared with analytical solutions and those obtained from the conventional finite element method (FEM) to evaluate the performance of the proposed model.

1. Introduction

Problems involving the nonlinear behaviour of saturated porous media are of considerable interest in geotechnical engineering. The bearing capacity of shallow foundations, slope stability problems, and many other boundary value problems lie in this category. To date, a large number of experimental, theoretical, and numerical works have been performed in this area. In terms of numerical analyses, the widely used finite element method (FEM) has been the major tool for the study of the behaviour of elasto-plastic materials [1–7]. However, the FEM suffers from inherent shortcomings, such as an overly stiff behaviour, strong reliance on the quality of the mesh, and problems related to mesh distortion in large deformation analysis.

Meshfree methods (MMs) have been proposed to overcome deficiencies associated with the FEM. The first MM, often referred to as smoothed particle hydrodynamics (SPH) [8,9], was introduced in the 1970s to solve problems in astrophysics. SPH was followed by a number of improved MMs in the 1990s, including the diffuse element method (DEM) [10], element-free Galerkin methods (EFGM) [11] and reproducing kernel particle methods (RKPM) [12]. Many of these MMs have already been utilised for solving coupled hydro-mechanical problems [13,14]. In the 2000s, the point interpolation methods (Polynomial PIM and radial PIM) were formulated by Liu and Gu [15,16]. Despite their simplicity and many advantages over other MMs [17–21], PIMs are not theoretically rigorous because of the problems related to discontinuity of the approximation function in the problem domain. To overcome this difficulty, a novel category of MMs based on PIMs were proposed [22–25]. In this approach, the generalised gradient smoothing technique is applied to the polynomial PIM and the radial PIM, resulting in a new class of MMs known as the Smoothed PIM (SPIM) and Smoothed RPIM (SRPIM). In these methods, the problem associated with the incompatibility of the approximation function is circumvented by adopting a constructed, rather than a compatible, strain field and therefore removing the need for calculation of the derivation of the shape functions. These methods are in essence a combination of MMs and the FEM, therein benefiting from the specific strengths of each method. In SPIMs, a background mesh remains required; however, unlike the FEM, the numerical solution is not heavily dependent on the quality of the background mesh, and a simple triangular mesh (in 2D problems) or tetrahedron mesh (in 3D problems) is often sufficient to ensure accuracy of the numerical solutions.

SPIMs are very efficient and have been applied in various fields of engineering such as solid mechanics [22,26,27], heat transfer [28], and, recently, soil mechanics [29,30] assuming linearly elastic material behaviour. However, applications of SPIMs to fully coupled hydro-mechanical problems in elasto-plastic porous media have so far been limited. Zhang et al. [33] applied SPIM to elasto-plastic analysis of two-dimensional problems with gradient-dependent plasticity, albeit for single-phase materials only. Soares [31,32] applied SPIM to the modelling of dynamic elasto-plastic

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problems in single-phase and two-phase materials; however, he adopted an approximation technique for the calculation of the coupling matrix of the discretised system of equations in the sense that he used Gauss points located on the boundary of the smoothing domains, rather than conventional Gauss points, for the calculation of the area integrations over the smoothing domains. This approach may introduce errors in the calculations, which can be controlled only by refining the background mesh, because adopting more Gauss points for the area integrations is not practical in the approach proposed by Soares [31,32]. To date, SPIM has not been rigorously applied to analysis of coupled flow and deformation in porous media subjected to elasto-plastic behaviour.

In this paper, an edge-based SPIM (ESPIM) is presented for coupled flow-deformation analyses of porous media assuming an elasto-plastic behaviour for the solid skeleton. The problem domain is divided into three-node triangular background cells through a Delaunay triangulation. Edge-based smoothing domains are then constructed using the cells of the triangular background mesh. The displacement field is constructed by employing the polynomial and radial PIMs, which possess the Kronecker delta property facilitating the imposition of the essential boundary conditions. The smoothed strain field is constructed by applying the smoothing operation technique over the smoothing domains. To ensure non-singularity of the moment matrix in the derivation of the nodal shape functions, two node selection schemes, referred to as Tr3 and Tr2L, are used to select the support nodes for shape function construction at each point of interest. The conventional Gauss points inside the smoothing domains are also used for the calculation of the compressibility and coupling matrices [29]. A substepping scheme [34] assuming known strain increments is utilised for stress integration, and the nonlinear system of equations is solved by adopting the modified Newton-Raphson iteration scheme. Numerical examples are presented to demonstrate the accuracy and application of the proposed formulation.

2. Governing equations

The general equations governing flow and deformation in a saturated deforming porous medium under the assumptions of small strains and negligible inertial forces are expressed as follows [35]

$$\mathbf{L}_d^{\mathrm{T}}\boldsymbol{\sigma} + \rho\mathbf{g} = \mathbf{0} \tag{1}$$

$$\mathrm{div}\left[\frac{\mathbf{k}_{\mathrm{f}}}{\mu_{\mathrm{f}}}(\nabla\tilde{p}_{\mathrm{f}} + \rho_{\mathrm{f}}\bar{\mathbf{g}})\right] - a_{\mathrm{f}}\dot{\tilde{p}}_{\mathrm{f}} + \mathrm{div}(\dot{\tilde{\mathbf{u}}}) = 0 \tag{2}$$

where $\mathbf{L}_{\mathrm{d}}$ is the differentiation matrix defined as

$$\mathbf{L}_{\mathrm{d}} = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 \\ 0 & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix} \tag{3}$$

with x_1 and x_2 being space coordinates. ∇ is the gradient operator matrix defined as $\nabla = \mathbf{L}_d^{\mathrm{T}}\boldsymbol{\delta}$, with $\boldsymbol{\delta} = [1 \quad 1 \quad 0]^{\mathrm{T}}$, and div stands for the divergence operator. Eq. (1) states the equilibrium of the soil water mixture, and Eq. (2) is the combination of the mass balance equation for the fluid phase with Darcy's law for the fluid flow in porous media. In these equations, bold imprints denote vectors and matrices, and the over-dot indicates time derivative. $\boldsymbol{\sigma}$ is the total stress vector; $\mathbf{g} = [0 \quad g \quad 0]^{\mathrm{T}}$ and $\bar{\mathbf{g}} = [0 \quad g]^{\mathrm{T}}$ are the gravity acceleration vectors, with g being the gravitational constant; $\tilde{\mathbf{u}}$ is the displacement vector of the soil skeleton; $\tilde{p}_{\mathrm{f}}$ is the fluid pressure (the tilde indicates a continuous field as opposed to the discretised approximation field introduced later); $\mathbf{k}_{\mathrm{f}}$ indicates the intrinsic permeability; μ_{f} is the dynamic viscosity of the fluid phase; ρ_{f} is the density of the fluid; and ρ is the porous medium density. $a_{\mathrm{f}} = n(c_{\mathrm{f}} - c_{\mathrm{s}}) + \eta c_{\mathrm{s}}$, where $\eta = 1 - \frac{c_{\mathrm{s}}}{c}$, n is the porosity, and c_{f}, c_{s} and c are the compressibility of the fluid phase, compressibility of the solid grains, and drained compressibility of the solid skeleton, respectively.

The other relationships used to complete Eqs. (1) and (2) are Terzaghi's effective stress principle, the incremental elasto-plastic constitutive law, and the relationship for small strains for the solid matrix, as shown in Eqs. (4), (5) and (6), respectively,

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \eta\tilde{p}_{\mathrm{f}}\boldsymbol{\delta} \tag{4}$$

$$d\boldsymbol{\sigma}' = \mathbf{D}^{\mathrm{ep}}d\boldsymbol{\varepsilon} \tag{5}$$

$$d\boldsymbol{\varepsilon} = \mathbf{L}_{\mathrm{d}}(d\tilde{\mathbf{u}}) \tag{6}$$

Note that $\boldsymbol{\sigma}'$ is the effective stress vector, and $\mathbf{D}^{\mathrm{ep}}$ is the tangent elasto-plastic constitutive matrix. For the soil skeleton, compression is taken as negative, and tension is taken as positive.

3. Edge-based smoothed point interpolation method

3.1. Function approximation

Point interpolation methods (PIM and RPIM) [15,16] are considered for the function approximation in this work. The first group of shape functions applied in the ESPIM are the polynomial point interpolation shape functions in which polynomials are used as the basis functions. The displacement field is approximated at the point of interest $\mathbf{x} = [x_1 \quad x_2]$ as

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{p} P_i(\mathbf{x})a_i = \mathbf{P}^{\mathrm{T}}(\mathbf{x})\mathbf{a} \tag{7}$$

where $\mathbf{u}(\mathbf{x})$ is the approximated displacement vector, $P_i(\mathbf{x})$ are the polynomial basis functions obtained from the Pascal's triangle of monomials for 2D problems, and $\mathbf{a}$ is a coefficient vector with yet unknown entries as follows:

$$\mathbf{P}^{\mathrm{T}}(\mathbf{x}) = [1 \quad x_1 \quad x_2 \quad x_1^2 \quad x_1x_2 \quad x_2^2 \quad \cdots] \tag{8}$$

$$\mathbf{a}^{\mathrm{T}} = [a_1 \quad a_2 \ldots \quad a_p] \tag{9}$$

where p is the number of supporting nodes for the point of interest.

The radial point interpolation shape functions based on radial basis functions (RBFs) are used in the ESRPIM. The approximated field function based on RPIM interpolation enriched with polynomials can be written as

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{p} R_i(\mathbf{x})a_i + \sum_{j=1}^{l} P_j(\mathbf{x})b_j = \mathbf{R}^{\mathrm{T}}(\mathbf{x})\mathbf{a} + \mathbf{P}^{\mathrm{T}}(\mathbf{x})\mathbf{b} \tag{10}$$

where $R_i(\mathbf{x})$ and $P_j(\mathbf{x})$ are radial and polynomial basis functions, respectively; p is the number of supporting nodes for the point of interest; and l is the number of monomials used in the polynomial basis functions. It should be noted that a minimum of three monomials are required to ensure linear consistency (i.e., $l \geqslant 3$). Adding polynomials to the RPIM shape functions generally improves the accuracy of the results and interpolation stability of the nodal shape functions [36].

Among various RBFs available in the literature [37,38], the multi-quadrics (MQ) RBFs are chosen in the current study due to their simplicity and stability. Thus, $R_i(\mathbf{x})$ is expressed as

$$R_i(\mathbf{x}) = \left(r_i^2 + (\alpha_c d_c)^2\right)^q, \quad \alpha_c \geqslant 0 \tag{11}$$

in which r_i is the distance between the point of interest $\mathbf{x}$ and the node at $\mathbf{x}_i$, d_c is the local average nodal spacing, and α_c and q are the shape parameters, taken as 4.0 and 1.03 respectively, following the recommendations in [23,39].