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Research Paper

A novel nonlinear solution for the polygon scaled boundary finite element method and its application to geotechnical structures

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ABSTRACT

The polygon scaled boundary finite element method is semi-analytical and known for its high precision. However, the material nonlinearity cannot be maintained because this method uses an analytical solution in the radial direction. In this paper, a novel nonlinear algorithm is developed by introducing internal Gaussian points over a subdomain. The response of nonlinearity for a concrete-faced rockfill dam is modeled. The results correspond well with the results from finite element modelling, which demonstrates the method can be used to describe the nonlinear characteristics of geomaterials. Furthermore, this method offers promising flexibility for analyzing complex geometries without decreasing the precision.

1. Introduction

Material nonlinearity is an inevitable problem in the numerical simulations of many engineering structures. Many researchers are interested in studying and implementing nonlinear calculation methods and software. The finite element method (FEM) has been used extensively as an effective technique for obtaining reasonable solutions. In traditional FEM, the domain is often discretized into triangles or quadrilaterals. Polygon finite elements and related methods have been proposed in recent years. Some representative numerical methods are based on polygon elements, such as the rational function interpolation by Wachspress [1–3], the Voronoi cell finite element method (VCFEM) and polygon elements using natural neighbor shape functions [4,5], the planar arbitrary polygonal element method [6,7], the macro-element Galerkin method (MEGM) [8], the conforming polygonal finite element (CPFE) method [9] and the natural element method (NEM) [10]. Using polygon elements, the discretization becomes more flexible, offering high accuracy and a high degree of convergence, and the mechanical properties can be simulated conveniently and effectively. The aforementioned methods based on polygon finite elements [11–13] are promising in the numerical simulation field.

The scaled boundary finite element method (SBFEM), presented by Wolf and Song [14–17], combines the advantages of the boundary element method (BEM) and FEM. SBFEM is a semi-analytical method, in which a numerical solution is obtained in the circumferential direction and an analytical value can be derived in the radial direction. Compared with conventional FEM, SBFEM increases the degree of precision and convergence in the numerical simulations and significantly reduces the number of degrees of freedom in the computational model. In addition, no fundamental solutions are require for the application of the variational principle. These advantages/benefits make SBFEM a powerful tool for a wide variety of linear elasticity problems.

During the past two decades, SBFEM has been used to many problems in engineering fields, such as those related to unbounded media [18], electrostatic fields [19], electromagnetic waveguides [20], magneto-electro-elastic plates [21] and fluid-structure interactions [22]. In unbounded media, the radiation conditions at infinity can be satisfied exactly and automatically. The number of elements is dramatically reduced in the fluid-structure interaction field because discretization is only performed at the boundary. In electromagnetic problems, the singularity of the contact between different materials can be efficiently solved.

The polygonal scaled boundary finite element method (PSBFEM) was recently developed based on SBFEM. The concept of polygons was introduced to handle more complex geometries, making SBFEM more flexible and convenient. Compared with

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FEMs, PSBFEM contributes to getting a higher degree of precision and convergence [23]. Many researchers have implemented PSBFEM through numerical simulation. Ooi [24–27] applied this theory to solve the fracture analysis of structures; Bao [28] simulated the crack propagation of a gravity dam under seismic loading; and Mingguang [29] modeled the crack propagation for contacts between different materials.

However, SBFEM are mostly subject to linear elastic materials and rarely used in non-linearity materials in practice, especially in engineering fields. Combining SBFEM with the homotopy analysis method (HAM), Lin and Liao [30] applied SBFEM to solve nonlinear problems. The least squares method was introduced to adjust to the integration polynomial to simulate plasticity over each polygon, and the PSBFEM formulation for elasto-plastic analysis was derived by Ooi et al. [31]. This study presents a different approach for applying SBFEM to non-linear materials.

According to the integral rule of triangles in FEM, three internal Gaussian integration points are introduced over a sector covered by a line element on the boundary, and a novel nonlinear polygon scaled boundary finite element method (NPSBFEM) is developed to simulate the nonlinear properties of geomaterials. The elasto-plastic constitutive matrix, stiffness matrix and imbalance force vector can be conveniently derived by combining the development platform of the finite element procedure. Then, SBFEM can be used to model nonlinear material properties with polygon elements.

The remainder of this paper is organized as follows. The basic theoretical derivation of the PSBFEM is introduced in Section 2. The displacement field, the shape function of the polygon elements and the incremental strain and stress fields are also described in Section 2. Section 3 describes the scaled boundary polygon formulation for elasto-plastic analysis. The development platform of the proposed algorithm is introduced in Section 4. The reliability of the procedure is validated by two numerical examples in Section 5. Section 6 summarizes the major conclusions that can be drawn from this study.

2. Theoretical derivation of the PSBFEM

2.1. Coordinate transformation

An arbitrary domain Ω can be discretized with a mesh of arbitrary n-sided polygons (where n can be any integer $\geqslant 3$). Any polygon can be treated as an SBFEM subdomain as long as its geometry satisfies the SBFEM scaling requirement, i.e., any point on the polygon boundary must be directly visible from the scaling center. The numerical results of the domain are obtained after solving each subdomain with the SBFEM.

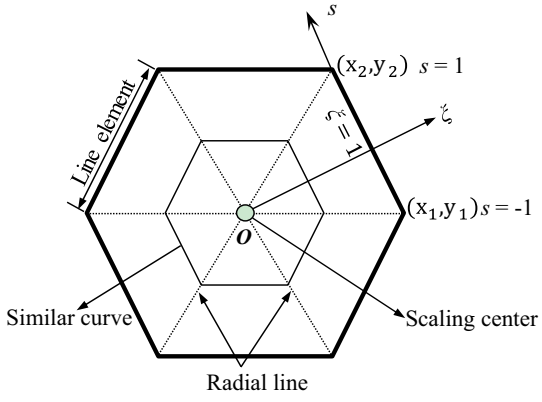

Fig. 1. Polygon representation of the SBFEM.

Fig. 1 shows a typical polygon modeled using the PSBFEM. A scaling center is defined at the geometric center of the polygon. Each edge on the polygon is discretized using one-dimensional line elements with a local coordinate s that varies from -1 to +1, and a radial coordinate ξ is defined that varies from zero at the scaling center to unity on the boundary. Two-node linear elements are used in this study; the Cartesian coordinates of an element can be expressed in terms of the scaled boundary coordinates as follows:

$$\boldsymbol{x}_b(s) = \boldsymbol{N}(s)\mathbf{x}_b \tag{2-1}$$

$$\boldsymbol{y}_b(s) = \boldsymbol{N}(s)\mathbf{y}_b \tag{2-2}$$

$$\boldsymbol{N}(s) = [N_1(s), N_2(s), N_3(s), \ldots N_m(s)] \tag{2-3}$$

where $\mathbf{x}_b$ and $\mathbf{y}_b$ represent the vector of nodal coordinates on the boundary, $\boldsymbol{x}_b(s)$ and $\boldsymbol{y}_b(s)$ are the coordinate vectors along the line element, and $\boldsymbol{N}(s)$ is the shape function vector of a line element with m nodes on the polygon boundary. Only the boundary is discretized with line elements in each subdomain of SBFEM, so increasing the order of the shape function does not increase the mesh complexity. Increasing the order of the shape functions is convenient if necessary, and standard one-dimensional Gauss-Lobatto-Lagrange shape functions can be used. The entire polygon domain can be captured by scaling the polygon boundaries according to the radial coordinate with respect to the scaling center $\boldsymbol{O}$. The scaled boundary coordinate system (ξ, s) is related to the Cartesian coordinate system by the scaling equation as follows:

$$x(\xi, s) = \xi\boldsymbol{N}(s)\mathbf{x}_b \tag{2-4}$$

$$y(\xi, s) = \xi\boldsymbol{N}(s)\mathbf{y}_b \tag{2-5}$$

where (x, y) is the geometry of a point inside the domain.

2.2. Scaled boundary polygon shape functions

For a sector covered by a line element on a polygon boundary, an approximate solution for the displacement at any point in a subdomain can be written in a form related to the local coordinates of the SBFEM:

$$\boldsymbol{u}(\xi, s) = \boldsymbol{N}_u(s)\boldsymbol{u}(\xi) \tag{2-6}$$

where the nodal displacement functions $\boldsymbol{u}(\xi)$ are introduced, which denote the displacements along the radial lines are analyzed with respect to the radial coordinate, and $\boldsymbol{N}_u(s)$ represents the shape function matrix, which has the following form:

$$\boldsymbol{N}_u(s) = \begin{bmatrix} N_1(s) & 0 & N_2(s) & 0 & \cdots & 0 & N_m(s) & 0 \\ 0 & N_1(s) & 0 & N_2(s) & 0 & \cdots & 0 & N_m(s) \end{bmatrix} \tag{2-7}$$

Then, the partial differential equations of equilibrium for a polygon derived from the Galerkin weighted residual method result in the following equation, and the radial displacement functions $\boldsymbol{u}(\xi)$ are the solution to the SBFEM governing equation for displacement.

$$\boldsymbol{E}_0\xi^2\boldsymbol{u}(\xi)_{,\xi\xi} + (\boldsymbol{E}_0 - \boldsymbol{E}_1 + \boldsymbol{E}_1^T)\xi\boldsymbol{u}(\xi)_{,\xi} - \boldsymbol{E}_2\boldsymbol{u}(\xi) + \boldsymbol{F}(\xi) = 0 \tag{2-8}$$

where the coefficient matrices $\boldsymbol{E}_i (i = 0, 1, 2)$ depend only on the geometry and material properties of the subdomain, which are evaluated for line elements and assembled over the discretized polygon boundary, and $\boldsymbol{F}(\xi)$ is a load vector that includes contributions from side-face traction, body and thermal loads. Introducing a new vector $\boldsymbol{X}(\xi)$ when the load vector $\boldsymbol{F}(\xi)$ becomes zero transforms Eqs. (2-8) into a first-order homogeneous differential equation system with respect to ξ: