Computers and Geotechnics 84 (2017) 183–197

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Head-based isogeometric analysis of transient flow in unsaturated soils

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ARTICLE INFO

Article history:
Received 24 April 2016
Received in revised form 22 November 2016
Accepted 24 November 2016

Keywords:
Unsaturated soil
Isogeometric analysis
Heterogeneous soils
Transient flow
Finite element method

ABSTRACT

An isogeometric analysis (IGA) is introduced to obtain a head-based solution to Richards equation for unsaturated flow in porous media. IGA uses Non-Uniform Rational B-Spline (NURBS) as shape functions, which provide a higher level of inter-element continuity in comparison with Lagrange shape functions. The semi-discrete nonlinear algebraic equations are solved using a combination of implicit backward-Euler time-integration and Newton-Raphson scheme. The time-step size is adaptively controlled based on the rate of changes in the pore pressure. The results from the proposed formulation are compared and verified against an analytical solution for one-dimensional transient unsaturated flow in a homogenous soil column. The proposed method is then applied to four more complex problems including two-dimensional unsaturated flow in a two-layered soil and a semi-circular furrow. The test cases in two-layered soil system involve sharp variations in the pressure gradient at the intersection of the two media, where the pore water pressure abruptly changes. It is shown that the proposed head-based IGA is able to properly simulate changes in pore pressure at the soils interface using fewer degrees of freedom and higher orders of approximation in comparison with the conventional finite element method.

1. Introduction

Transient flow in variably saturated porous media is a common interest in many research studies. Its application covers a high range of disciplines from soil science and subsurface hydrology to material research for industrial porous media, geotechnical and petroleum engineering [1]. In geotechnical engineering, fluid flow in unsaturated soils has shown to play a controlling role in various practical problems including slope stability [2–5], lateral earth pressure [6], reinforced soil structures [7], and bearing capacity of foundations [8,9]. Loss of suction and the subsequent reduction in soil's effective stress, due to steady or transient flow can adversely impact the performance and integrity of variably saturated slopes and earthen structures [10,11].

From physical point of view, the air pressure can be assumed constant and fluid transportation is simply governed by Richards' equation which is the combination of generalized Darcy's law and continuity [12,13]. Richards' equation is considerably nonlinear because of highly variable effective conductivity that varies from very dry to fully saturated conditions. This variation results in a nonlinear relation between negative pore pressure and degree of saturation. The nonlinear nature of transient flow in unsaturated medium restricts the analytical solutions to few problems with simple boundary conditions [14–20]. For general unsaturated problems, numerical methods are more applicable than analytical methods. Numerical methods that represent Richards' equation can be classified into three general forms of continuous equations: moisture-based, head-based, and mixed form.

Moisture-based (θ-based) formulation represents the governing equation in terms of moisture content. The moisture form generally performs very well when implemented in an iterative procedure and allows the use of large time steps. However, this form is only applicable to strictly unsaturated and homogenous conditions [1]. Head-based (h-based) form formulates the governing equation based on pressure head. This form is applicable to both saturated and unsaturated conditions and it can be used to model heterogeneous media as well. Nevertheless, its performance encounters difficulties, especially for problems involving infiltration into very dry soils. Very short time steps are usually implemented to prevent divergence in the iterative solutions [1]. In an attempt to take advantages of both moisture- and head-based frameworks, the primary variable switching technique has been proposed [21,22]. This technique uses the moisture content as the primary variable when the domain is partially saturated and

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http://dx.doi.org/10.1016/j.compgeo.2016.11.018
0266-352X/

the pressure head as the primary variable when the domain is fully saturated. Despite the simplicity of the concept, there is still the possibility of divergence during the iterative procedure. For instance, in the Finite Element Method (FEM), the solution procedure may diverge if a given node with a degree of saturation below the switching criteria changes to fully saturated conditions during subsequent iterations. This problem is intensified as mesh is refined and wetting front covers a greater number of nodes in the associated time step [13]. Moreover, with respect to any form of governing equation, the efficiency and robustness of the results are highly influenced by spatial and temporal discretization methods as well as linearization methods for nonlinear equations. For spatial discretization, the finite difference method [23–25], FEM [26–32], and the finite volume method [33,34] are commonly used numerical methods while finite deference is usually reserved for time discretization. For solving nonlinear equations, Picard and Newton-Raphson methods are popular schemes which are vastly used in transient flow simulations in porous medium [1,35,36].

While conventional numerical methods are successful in the simulation of transient flow problems in variably saturated soils, it is still desirable to develop more efficient analysis methods. In recent years, Isogeometric analysis (IGA) has been increasingly employed in variety of engineering fields like high-performance computing (PetIGA: A Framework For High-Performance Isogeometric Analysis) [37], plates [38–40], incompressibility [41], electromagnetics [42], phase fields [43,44], poroelasticity with fully saturated conditions [45], flow regime in shale [46], thermal buckling [47], and Cahn-Hilliard equation [48], among others. As discussed in the previous studies, IGA offers advantageous features including exactness of reproducing the geometry, higher-order continuity, and simpler mesh generation and mesh refinement procedures in comparison to alternative numerical methods. Extending IGA applications to different fields and examining advantages and disadvantages of using different types of splines in IGA have been increasingly investigated in recent years. For instance, non-uniform rational B-spline (NURBS), T-spline [50], PHT-splines [51], LR-splines [52] are different splines alternatives used in IGA. NURBS are well-known in computational geometry but they show inability to produce watertight geometries in complex configurations [53]. T-splines addresses this problem by creating a simple patch but it is shown that linear independence of the T-spline basis functions is not guaranteed [54,55]. Moreover, the complexity of knot insertion under adaptive refinement is considerable for T-splines [53].

Nguyen et al. [49] utilized IGA in unsaturated flow problems in the moisture-based form and introduced a successful framework including NURBS basis for spatial discretization and the implicit backward-Euler method for time discretization. However, the proposed solution is limited to homogenous problems and their solution is unable to simulate mixed unsaturated-saturated conditions. Moreover, they used constant time steps in the time integration scheme which is not computationally cost effective.

In the present study, we propose a head-based method implementing IGA to solve transient flow in heterogeneous unsaturated soils. This numerical approach utilizes NURBS basis functions for spatial discretization which benefits from the high-order continuity of IGA interpolation. The implicit backward-Euler method with adaptive time-stepping is used for time marching. This technique utilizes larger time step sizes where the rate of changes in the pore pressure is not significant, while decreasing the time step size when the changes in pore pressure are considerable and affect the solution convergence. In order to avoid oscillation at the wetting front, the lumped mass matrix technique is used for numerical integration [1,25] and the Newton-Raphson method is employed to solve the nonlinear equations.

The rest of the paper is organized as follow: Section 2 describes the methodology of the presented study, in which the governing equation, variational statement, IGA spatial discretization, and time discretization are explained. In Section 3, the proposed method is benchmarked against a one dimensional (1D) analytical solution representing homogenous soil [56]. The application of the proposed IGA method is further extended in Section 4 which presents the implementation of higher order IGA in a highly nonlinear problem (referred to as Celia et al.'s problem [25]) and the comparison with an alternative FEM solution [1,25]. Then numerical solution is implemented to a semi-circular furrow under high rate of infiltration. The accuracy and applicability of the method for heterogeneous medium is investigated through two numerical examples. The first example shows a 1D two-layer soil system (referred to as Brunone et al.'s problem [57]) and results are compared with those from quadratic Lagrangian FEM. The second example represents a two dimensional (2D), two-layered soil subjected to infiltration. Finally, Section 5 highlights the main conclusions drawn from this study.

2. Methodology

2.1. Governing equations

Since Richards' equation employs a single equation to describe the unsaturated flow, it requires the choice of primary and secondary variables. In the h-based formulation, the head is introduced as the primary variable and the multidimensional generalization of the governing equation is expressed as:

$$\left(\frac{\theta}{\phi}S_o + C(\psi)\right)\frac{\partial h}{\partial t} = \nabla \cdot [k_r(\psi)K\nabla h] \tag{1}$$

where θ is the moisture content, ϕ is the porosity, S_o is the specific storage coefficient, h is the total head, ψ is the pressure head, $k_r(\psi)$ is relative conductivity function, K is the tensor of saturated hydraulic conductivity, ∇ is the gradient operator, and $C(\psi)$ is the moisture capacity function. In this work, a general form of K is considered enabling simulation of isotropic/anisotropic and homogenous/heterogeneous soils.

After obtaining the primary variable (h) from Eq. (1), ψ can be easily calculated based on classical soil mechanics concepts. Subsequently, a parameterized retention curve (e.g., Gardner [58], or van Genuchten and Mualem (VGM) [59,60]) can be used to introduce θ, k_r, and C functions:

$$\text{Gardner}: \quad \theta(\psi) = \theta_r + (\theta_s - \theta_r)e^{-\alpha_G\psi} \tag{2a}$$

$$\text{VGM}: \quad \theta(\psi) = \begin{cases} \theta_r + (\theta_s - \theta_r)(1 + |\alpha_V\psi|^n)^{-m} & \psi < 0 \\ \theta_s & \psi \geqslant 0 \end{cases} \tag{2b}$$

Effective degree of saturation (S_e) is defined based Eq. (2a) or (2b) as:

$$S_e = \frac{\theta(\psi) - \theta_r}{\theta_s - \theta_r} \tag{3}$$

Subsequently the relative hydraulic conductivity is defined:

$$\text{Gardner}: \quad k_r(\psi) = e^{-\alpha_G\psi} \tag{4a}$$

$$\text{VGM}: \quad k_r(S_e) = S_e^{1/2}\left[1 - \left(1 - S_e^{\frac{1}{m}}\right)^m\right]^2 \tag{4b}$$

where θ_r and θ_s are residual and saturated moisture contents, respectively, α_G is Gardner's curve fitting coefficient, α_V and n are VGM's fitting parameters, and m can be assumed as $m = 1 - \frac{1}{n}$.

Finally, the moisture capacity function can be obtained by introducing the first derivative of Eq. (2):