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Research Paper

# Coupled consolidation in unsaturated soils: From a conceptual model to applications in boundary value problems

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ABSTRACT

The paper presents the Finite Element formulation of the equations proposed by Tsiampousi et al. (2016) for coupled consolidation in unsaturated soils. Their coupling is discussed in relation to a conceptual model which divides soil behaviour into zones ranging from fully saturated to dry states. The numerical simulation of a laboratory experiment involving drainage of water from a vertical column of sand is used to validate the equations. Finally, the example of rainfall infiltration into a cut slope highlights how aspects of the conceptual model are reflected in the numerical analysis of boundary value problems involving unsaturated soils.



## 1. Introduction

Numerical analysis of various geotechnical problems involving hydro-mechanical coupling in unsaturated soils is now becoming progressively more common as the relevance of unsaturated soil mechanics to a number of engineering applications has started to be recognised (e.g. [2,15,14,32]). Compacted fills are by nature unsaturated at the time of compaction and depending on the loading and hydraulic boundary conditions imposed may remain unsaturated for years after their construction. Other examples refer to unsaturated soils naturally occurring above the groundwater table in arid and semi-arid parts of the world. Processes like consolidation and swelling due to loading and unloading and seasonal pore water pressure variations (e.g. due to the combined effect of rainfall and evapotranspiration) may affect the mechanical behaviour of geotechnical structures, such as cut slopes, embankments, foundations and pavements. As there are no analytical solutions for the hydro-mechanical coupling in unsaturated soils, numerical techniques, such as the Finite Element (FE) method, need to be employed. For such an approach to be possible the equations governing coupled consolidation need to be implemented in a numerical code and combined with constitutive, soil-water retention (SWR) curve and permeability models, while applying appropriate mechanical and hydraulic boundary conditions.

The implementation of the governing equations proposed by Tsiampousi et al. [35] in the numerical code ICFEP [25] is presented herein. ICFEP incorporates a number of constitutive (e.g. [16,33] and SWR models [34] to simulate unsaturated soil behaviour. Additionally, permeability may vary as a function of void ratio and suction [25], or degree of saturation [36], while desiccation under tensile principal stresses can also be modelled [23]. In addition to numerous mechanical and hydraulic boundary conditions (e.g. excavation, construction, compaction, infiltration, prescribed displacements and pore water pressures, tied degrees of freedom), ICFEP includes boundary conditions which simulate the effect of vegetation [24] and precipitation, which can be combined with an automatic incrementation algorithm [25,31].

ICFEP adopts a modified Newton-Raphson solution technique with an error controlled sub-stepping stress-point algorithm to approximate non-linear behaviour in a linear, step-wise fashion. In this way, although the governing equations were developed assuming linear elasticity, unsaturated soil behaviour can be modelled as non-linear. Non-linearity may arise from the constitutive model employed (stress-strain relationship), the relationship between permeability and suction (or degree of saturation), the SWR curve and the governing equations themselves. Indeed, Tsiampousi et al. [35] demonstrated that the additional parameters

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$\Omega$, $\omega$ and $H$, which are required to extend the governing equations to unsaturated soil states, vary with suction in a highly non-linear manner.

The governing equations implemented into ICFEP differ from others in the literature (e.g. [9,39]) in that a clear distinction is made between the two moduli controlling the effect of matric suction on direct strains and the effect of net stress on the volumetric water content. As a result, parameters $\Omega$, $\omega$ and $H$ mentioned above relate to four moduli rather than three, as in Wong et al. [39]. Tsiampousi et al. [35] show that $\Omega$, $\omega$ and $H$ can be wholly determined in a consistent manner albeit being dependent on the SWR and constitutive models used to reproduce soil behaviour. In particular, they show that different Equations correspond to different models. More specifically, if the SWR model accounts for the effect of specific volume $v$ on the degree of saturation, $S_r$, in addition to matric suction, $s$, i.e. $S_r$ is a function $f$ of both $v$ and $s$, $S_r = f(s, v)$, parameter $\Omega$ is no longer the same as in the case where $S_r$ is a function $f$ of $s$ only, $S_r = f(s)$. This is further discussed here in relation to the coupling of the governing equations. In particular, the discussion focuses on how the additional parameters $\Omega$, $\omega$ and $H$ enable the governing equations to reflect the mechanical and hydraulic changes that an element of soil undergoes as it progressively changes from a state of full saturation to completely dry conditions and back to full saturation. To aid the discussion the conceptual model of Tsiampousi et al. [35] is utilised.

Finally, the new governing equations are employed in two numerical studies: first, in the numerical simulation of a laboratory experiment involving drainage of water from a vertical column of sand, and subsequently in the numerical study of a cut slope subjected to rainfall infiltration. In the former analysis, the numerical results are compared to experimental data, but also to numerical results obtained by rigid unsaturated and coupled fully saturated analyses in order to highlight the difference in the simulated behaviour when approaches other than unsaturated coupled consolidation analysis are used. In the analysis of the cut slope, the same three types of analyses are compared in terms of their numerical results when the cut slope is subjected to rainfall of low intensity but long duration. The differences observed can be justified by the effect of parameters $\Omega$, $\omega$ and $H$, highlighting that aspects of the conceptual model are indeed reflected in the numerical analysis of boundary value problems involving unsaturated soils. Finally, the unsaturated analysis of the cut slope was repeated for a rainfall of very low intensity and long duration and a rainfall of high intensity and short duration. The results indicate that both intensity and duration of rainfall are important to slope stability, thus the analysis requires sophisticated boundary conditions to simulate rainfall realistically.

## 2. Governing equations and conceptual model

Tsiampousi et al. [35] show that the governing equations can take the following form:

$$\varepsilon_x = \frac{(\sigma_x - u_a)}{E} - \frac{\mu}{E}(\sigma_y + \sigma_z - 2u_a) + \left(\frac{u_a - u_w}{H}\right) \quad \& \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \tag{1}$$

(and similar for $\varepsilon_y$, $\varepsilon_z$, $\gamma_{yz}$ and $\gamma_{xz}$) for the soil skeleton and:

$$\theta_w = \Omega\varepsilon_{vol} + \omega(u_a - u_w) \tag{2}$$

for the water phase, where:

- $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_z$ are direct strains in x, y and z directions, respectively;
- $\gamma_{xy}$ is the shear strain acting on the x-plane in the y-direction (similar for $\gamma_{yz}$ and $\gamma_{xz}$);
- $\sigma_x$, $\sigma_y$ and $\sigma_z$ are the direct total stresses in the x, y and z directions, respectively;
- $\tau_{xy}$ is the shear stress acting on the x-plane in the y-direction (similar for $\tau_{yz}$ and $\tau_{xz}$);
- $\varepsilon_{vol}$ is the volumetric strain and $\theta_w$ is the volumetric water content;
- $u_a - u_w$ is the matrix suction, $s$, $u_a$ being the air pressure and $u_w$ being the water pressure;
- $E$ is Young's modulus of the soil structure and $\mu$ is Poisson's ratio;
- $G$ is the shear modulus, $G = E/2(1+\mu)$;
- $\Omega$, $\omega$ and $H$ are additional moduli in the governing equations for unsaturated soil states, which, as Tsiampousi et al. [35] demonstrate, are defined by the following general equations:

$$\Omega = -S_r - e\frac{\partial S_r}{\partial v} \tag{3}$$

$$\omega = n \cdot \frac{\partial S_r}{\partial s} \tag{4}$$

$$H = -\frac{3}{\frac{1}{v_0} \cdot \frac{\partial v}{\partial s}} \tag{5}$$

The three moduli need to satisfy the following equation:

$$\omega = \left(\frac{1}{R} - \frac{3\Omega}{H}\right) \tag{6}$$

where $1/R$ is the slope of the soil-water retention (SWR) curve in terms of volumetric water content, i.e. $1/R = \partial\theta_w/\partial s$.

For consistency, parameters $\Omega$, $\omega$ and $H$ need to be adjusted to the particular SWR and constitutive models employed to represent soil behaviour. Additionally, they are not constant but vary with suction. An example of their variation with suction is given in Fig. 1. This corresponds to Case 1b in Tsiampousi et al. [35].

To highlight the associated implications in terms of soil behaviour, Tsiampousi et al. [35] present a conceptual model which draws heavily on the work of White et al. [38], and divides the soil into the four principle zones. Zone 1 is the zone of full saturation (Fig. 2(a)). Within zone 2 air is present in the soil pores in the form of occluded bubbles, having come out of solution. Air may have also started to penetrate into the soil forming air-boundaries but will have not yet penetrated past the outer-most soil particles, as shown in Fig. 2(b). In zone 3, air will have penetrated significantly into the soil. Zone 3 can be further divided in two zones, A and B. Zone 3A distinguishes the situation where, although the air phase is continuous from any point at which it is present within the soil back to an air boundary, it is not continuous all the way across the element (Fig. 2(c)). On the contrary, the water phase is continuous across the element and free to flow in all directions. The switch to zone 3B occurs when the air phase becomes continuous across the element (see Fig. 2(d)), while the water phase is also still continuous. Note that Fig. 2(d) illustrates a two-dimensional slice through a three-dimensional element. Although in the figure it appears that the water phase has become discontinuous this is not actually the case. The switch from zone 3A to 3B is assumed to occur at the point of inflection of the SWR curve when plotted on a semi-logarithmic plane (see Fig. 3). Once the soil element is desaturated to the point that there can be no further flow of water, the model enters zone 4 (Fig. 2(e)). Tsiampousi et al. [35] demonstrate that the boundaries between the zones are shifted to the left (i.e. lower suctions) for wetting in comparison to drying, as a result of the hydraulic hysteresis (see for example Fig. 1 where the zones on drying and wetting have been marked). Explaining and modelling soil behaviour with reference to the degree of saturation rather than suction would avoid this complexity arising from the hydraulic hysteresis experienced by most unsaturated soils. Nonetheless, customarily, it is nodal pore water pressures which are the primary

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