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Efficient meso-scale homogenisation and statistical size effect analysis of concrete modelled by scaled boundary finite element polygons

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HIGHLIGHTS

- Develop a new homogenisation approach using SBFE polygons for meso-models of concrete.
- Demonstrate much improved accuracy and considerable reduction in DOFs over the FEM.
- Propose a Weibull-like size effect law of effective elastic moduli considering porosity.
- The critical RVE size is 4.5 times the maximum aggregate size.

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ABSTRACT

This study develops an efficient numerical homogenisation approach for meso-scale concrete samples with randomly generated and packed aggregates and pores. A simple algorithm is devised to discretize samples into meshes consisting of semi-analytical scaled boundary finite element (SBFE) polygons only. As each aggregate is modelled by one SBFE polygon and only polygonal boundaries are discretized into nodes, the degrees of freedom of a model is significantly reduced compared with conventional finite element models. The volumetrically averaged stress inside a SBFE polygon is semi-analytically integrated, leading to high accuracy in the homogenised elastic properties. The effects of model size and porosity are statistically studied by extensive Monte Carlo simulations. A size effect law taking porosity into account is proposed to predict effective elastic moduli in good agreement with experimental data up to 200 mm model size. The meso-models are found statistically homogeneous when the size is about 4.5 times the maximum aggregate size.

1. Introduction

Concrete is a composite material widely used in engineering structures. It is generally assumed homogenous for simplicity at the structural or macro scale. At the meso-scale, the material is highly heterogeneous, with aggregates and pores of various sizes and geometries randomly distributed in the homogeneous mortar [1–3]. The micro/meso-scale heterogeneous structures of concrete can be used to determine the macroscopic, effective properties such as elastic moduli and Poisson's ratio, i.e., homogenisation [2]. The homogenised effective properties may be used as engineering constants for structural design in the future, hopefully without the need of time-consuming experiments.

A number of homogenisation methods for heterogeneous materials, either analytical or numerical, are available in the literature. Analytical methods estimating bounds of effective properties were developed by Voigt [4], Reuss [5], Hill [6] and Hashin and Shtrikman [7]. Eshelby [8] gave solutions for the problem of a single ellipsoidal inclusion embedded in an infinite matrix. These solutions have been extended to many problems with assumptions of weak or no inter-phasic interactions [9]. Other popular analytical methods include the self-consistent method [10], the Mori-Tanaka method [11,12] and the Christensen model [13]. Most of the above analytical methods assume simple geometries for inclusions and often neglect or simplify the meso-scale inter-phasic interactions, and their applicability to concrete-like materials with complex meso-structures and interactions is thus limited. Numer-

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ical homogenisation methods, mostly based on the finite element method (FEM), are widely used instead [2,14–24]. These methods aim to obtain an effective constitutive relation by calculating volumetrically averaged field variables under a series of boundary conditions. The volume averaging takes place over a statistically representative sample, referred as a representative volume element (RVE). To obtain statistical data such as mean and standard deviation of effective properties, a large number of samples with randomly distributed inclusions and pores should be simulated for a model size, often by repetitive Monte Carlo simulations (MCSs) [25,26]. Another important but hardly-investigated problem in homogenisation is that, a heterogeneous model can be assumed statistically homogeneous only when its size is larger than a critical value, namely, the critical RVE size [27,28]. To determine this critical RVE size, again a large number of repetitive simulations for a range of sizes are needed. Therefore, high computational costs are expected, apart from sophisticated mesh generation for the complicated meso-structures of concrete and very fine meshes necessary to calculate accurate average stress, when the FEM is used. This can be further exacerbated if the randomly-distributed pores are modelled. Although it is known that the 2–6% porosity in concrete plays an important role in its mechanical properties [29], a quantitative, statistical understanding of the porosity effects is still not available.

To overcome the aforementioned difficulties in homogenisation of concrete using the analytical methods and the FEM, we herein propose a numerical homogenisation approach based on the scaled boundary finite element method (SBFEM). The SBFEM, developed by Wolf and Song [30], is a semi-analytical method combining the advantages of the FEM and the boundary element method (BEM) and also possessing its own merits. It discretizes the SBFE subdomain or polygon boundaries only and thus reduces the modelled spatial dimensions by one as the BEM, but it needs neither fundamental solutions nor singular integrations. Therefore, the simplicity in representing geometries as in the BEM and the wide applicability as in the FEM are both retained. In addition, the SBFEM is semi-analytical and thus has much higher accuracy than other methods for the same number of degrees of freedom (DOFs). It has proved very accurate and efficient for solving problems with singularities and unbounded domains [31–42]. For example, it has been applied to model linear and nonlinear crack propagation [31,32], dynamic fracture [33], elastoplastic fracture [34], fluid field problems [35,36], and soil-foundation problems under earthquake [37,38]. The advantages of polygonal SBFEM are further demonstrated in recent reports by integration of quadtree/octree meshing techniques [39,40], isogeometric analysis [41] and the extended finite element method [42]. However, the polygonal SBFEM has been extended for homogenisation of porous magneto-electroelastic materials only very recently [43].

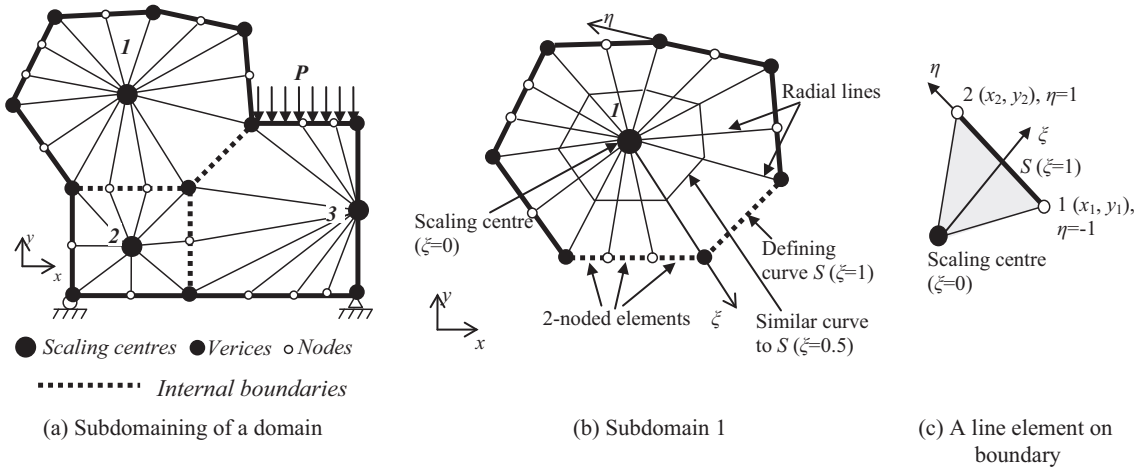
In this paper, we develop an efficient SBFE polygon-based approach for numerical homogenisation of meso-scale models of concrete with randomly-generated and packed aggregates. In this approach, each polygonal aggregate is naturally modelled by only one SBFE polygon without nodal discretization inside. This not only reduces DOFs, but makes automatic meshing of the meso-models simpler. The mortar matrix is discretized by SBFE polygons too, using the Delaunay triangulation method. The volumetrically averaged stress is derived from the SBFEM governing equations. Extensive Monte Carlo simulations are carried out with statistical analysis of the homogenised effective properties, from which the effects of model size and porosity are elucidated.

2. Methodology

2.1. The scaled boundary finite element method

The basic concept of the SBFEM is illustrated in Fig. 1 by an example. A domain of an arbitrary shape is shown in Fig. 1a and is divided into three subdomains or polygons. Any subdivision, with various numbers, shapes and sizes of subdomains, can be used as long as a scaling centre for each subdomain can be found from which the whole subdomain boundary is visible. In the SBFEM, each subdomain (Fig. 1b) is represented by scaling a defining boundary curve S relative to a scaling centre (x_0, y_0). The boundary curve is discretized using one-dimensional finite elements with local coordinate η that varies from −1 to +1. Although this study adopts two-noded linear elements shown in Fig. 1c, higher-order elements can be used to improve the solution's accuracy as in the FEM. The radial direction from the scaling centre to the boundary is described by a radial coordinate ξ with $\xi = 0$ at the scaling centre and $\xi = 1$ on S. The geometry of the subdomain is described by scaling the boundary curve S along ξ. It can be seen that η and ξ form a normalized local coordinate system similar to the polar coordinate system. The fundamentals of the SBFEM are given in many publications, e.g., [44–46], and only the key equations are given below for the convenience of discussion.

The governing equilibrium equations of the SBFEM have been derived within the context of virtual work principle for elastostatics by Deeks and Wolf [45]



(a) Subdomaining of a domain (b) Subdomain 1 (c) A line element on boundary

Fig. 1. Concept of the scaled boundary finite element method.