Construction and Building Materials 150 (2017) 851–859

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Constitutive modeling of the tensile and compressive deformation behavior of polyurea over a wide range of strain rates

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HIGHLIGHTS

- A visco-hyperelastic model is proposed based on thermodynamic derivation methods.
- The mechanical responses at several loading conditions are calculated by new model.
- Tensile and compressive behavior of polyurea at different strain rate is carried out.
- The results of polyurea at quasi-one dimensional strain state are predicted.

ARTICLE INFO

Article history:
Received 4 September 2016
Received in revised form 26 May 2017
Accepted 10 June 2017
Available online 19 June 2017

Keywords:
Polyurea
Rate-dependence
Constitutive behavior
Finite deformation
Visco-hyperelasticity

ABSTRACT

A three-dimensional visco-hyperelastic constitutive model is developed to describe the finite deformation mechanical behavior of polyurea materials at different strain rates. The constitutive model of finite strain visco-hyperelasticity is founded on the basis of the multiplicative decomposition of the deformation gradient tensor into hyperelastic and viscoelastic parts. The hyperelastic part uses the strain energy function to characterize the equilibrium response, and the viscoelastic part capturing the rate sensitivity uses the time partial derivative of strain energy function to characterize the time-dependent response. The nonlinear mechanical responses of the materials under several common loading conditions are calculated by the new proposed constitutive model. In order to validate the effectiveness of the constitutive model, the nonlinear stress-strain behavior of polyurea under uniaxial tension and compression are carried out in this paper. The experimental verification and the error analysis show that the model is capable of accurately representing the finite deformation stress-strain behavior of polyurea over a strain-rate range of 10^{-3}–10^{4}/s. In order to further broaden the application of the constitutive model, a method is presented which is available to predict the experimental results of polyurea at quasi-one dimensional strain state.

1. Introduction

In the event of an explosion, in addition to the primary blast effects, the speed of the debris can be up to 100 m/s. These constitute one of the major causes of casualties and injuries. The research showed that buildings, after spraying polyurea can maintain better structural integrity, leading to improved safety and reliability [1,2]. polyurea has also been used in the outer layer of armor plate of the high-mobility multipurpose wheeled vehicle, to reduce the damage of ballistic fragments and bullet [3]. In addition, polyurea as a coating or sandwich core material used in composite structures, can significantly improve the anti-impact capability of composite structures [4,5]. In order to fine-tune and improve the performance of elastomer-based protection systems, the systematic study on the finite deformation mechanical behavior of polyurea under different loading rates is needed for further design and optimization efforts.

Because of the complexity of the mechanical properties and microstructures of polyurea, the analysis of structures that utilize this material for high strain rate applications depends on numerical methods, whose accuracy is very much a function of the constitutive model used. Therefore, constitutive modeling of the mechanical behavior of polyurea for the purpose of integration in large scale finite element (FE) codes has drawn great attention from researchers recently. However, modeling of rate-dependent phenomena in elastomers is, perhaps, one of the most intricate tasks for the rheologists of present day [6,7]. Due to the presence of high deformability and strong nonlinearities, the constitutive model

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http://dx.doi.org/10.1016/j.conbuildmat.2017.06.055
0950-0618/

needs to be founded on finite strain theories that are also thermodynamically consistent. Green et al. [8] proposed a constitutive theory of finite viscoelasticity for modeling rate-dependent material behavior based on the multiplicative decomposition of the deformation gradient into elastic and inelastic parts. Subsequently, Sidoroff et al. and Haupt et al. [9,10] further developed simplified versions of this constitutive theory. However, they did not apply to experimental observations, and thus validity of their model is not proven. Reese [11] also developed a material model for the thermo-viscoelastic behavior of rubber-like polymers based on transient network theory. A main element of some of the approaches presented in the literature is to split the free energy of the system additively into equilibrium and non-equilibrium parts giving the elastic equilibrium stress and the viscosity-induced overstress based on the theory of Huber et al. [12]. Attard [13] also proposed a free energy density of the constitutive system for elastomers which consisted of the general Mooney expression for higher order elasticity for the incompressibility component and a generalization of the Simo and Pister [14] proposal for the compressibility component. All these models could get a better physical insight into the material if the driving motivations behind developing these functions could be evaluated based on experimental observations. It is well known that a good constitutive model is one that can give good comparison with experimental results for any stress state with one set of material parameters, gives stable results for all loadings, is applicable to a wide range of materials, and can be used to derive the constitutive relationship for a chosen stress tensor in general coordinates [15,19]. Based on this criterion, and following the theories used in earlier works of Attard et al. and Guo et al. [15,16], a suitable visco-hyperelasticity constitutive model is proposed to study the finite deformation mechanical behavior of polyurea over a wide range of strain rates.

The nonlinear tensile and compressive mechanical behavior of polyurea at different strain rates is studied by theoretical analysis and experimental research in this paper. Based on the second law of thermodynamics and mathematical derivation methods, a visco-hyperelastic constitutive model describing the finite deformation behavior of polyurea is proposed, which includes the effect of strain rate. In order to discuss the capability of the developed theory, the stress-strain responses simulated by the model have been compared with experimental data from Treloar and Kawabata et al. for rubber [20,21]. Finally, the resulting finite strain visco-hyperelastic model is used to simulate the mechanical response of polyurea materials under several common loading conditions.

2. Constitutive model

Due to the presence of high deformability and strong nonlinearities of the polyurea elastomer, the constitutive model needs to be founded on finite strain theories consistent with the natural laws of thermodynamics. Therefore, the phenomena observed under large strains require a model designed for finite strain viscoelasticity. This section introduces a model of this type that follows from the concept of the second law of thermodynamics. It is based on the multiplicative decomposition of the deformation gradient and the additive split of the free energy as introduced by Lubliner et al. [22]. Finally, a visco-hyperelastic constitutive model for polyurea loaded at different strain rates will be presented. Based on the microscopic mechanics theory of Noll [30], the constitutive equation should satisfy the three principles: (i) the stress in time t of a particle α in the material is uniquely determined by the deformation history of the whole material; (ii) The particle motion far from the particle α does not affect the stress on the particle α; (iii) the nature of matter has nothing to do with the choice of time and space coordinate system. According to the above principles, the constitutive functional of nonlinear elastic large deformation materials can be written as follows

$$\sigma(t) = \Psi_{i=-\infty}^{t}(E(i)) \tag{1}$$

where σ (t) is the Cauchy stress tensor in time t, and E(i) is the strain history tensor. The Taylor series expansion of the constitutive function is carried out in the vicinity of the constant strain history E. Let s = t − i, that is

$$\begin{aligned}\sigma(t) &= \Psi_{i=-\infty}^{t}(E(i)) = \Psi_{i=-\infty}^{t}(E(t-s)) \\ &= \Psi(E(t)) + \delta\Psi(E(t)|\delta(E(t-s)-E(t))) + o\|\delta(E(t-s)-E(t))\|\end{aligned} \tag{2}$$

where Ψ(E(t)) is the stress tensor function caused by current strain, and $\delta\Psi$(E(t)|δ(E(t−s) − E(t))) is the stress tensor function caused by strain difference history. When the history of strain difference is small relative to the passed time, the approximate constitutive relation of large deformation materials can be given by

$$\sigma(t) = \Psi(E(t)) + \delta\Psi(E(t)|\delta(E(t-s)-E(t))) \tag{3}$$

The hydrostatic pressure p is added to Eq. (3). The stress response of the visco-hyperelastic model under the loading condition can be finally represented as

$$\begin{aligned}\sigma &== -pI + S = -pI + S_e + S_v \\ &= -pI + \Psi(E(t)) + \delta\Psi(E(t)|\delta(E(t-s)-E(t)))\end{aligned} \tag{4}$$

where p is the hydrostatic pressure which needs to be determined from the boundary conditions of the problem under consideration, I is the unit tensor, S is the Cauchy deviatoric stress tensor, S_e is the deviatoric stress tensor caused by current strain, i.e. hyperelastic deformation part; S_v is the deviatoric stress tensor caused by strain difference history, i.e. viscoelastic deformation part. The above deduction is consistent with the stress decomposition of Yang et al. and Ju et al. [17,18].

2.1. Basic theory

In finite strain kinematics, the local mapping between the initial and current configuration of a deformable body under motion is described by the deformation gradient tensor F

$$F = \partial x/\partial X = \sum_{i=1}^{3}\lambda_i n_i \otimes N_i \tag{5}$$

where X is the coordinate of a typical particle in the reference configuration and refers to a given Cartesian coordinate system; x is the coordinate of a typical particle in the current configuration, λ_i is the deformation in the three principal directions, and $\lambda_i = 1 + \varepsilon_i$, N_i and n_i are the material and spatial vector triads. The left Cauchy-Green tensor B describes the finite deformation of the materials, and I_1, I_2 and I_3 are the three principal invariants of B

$$B = FF^{T} = \sum_{i=1}^{3}\lambda_i^2 n_i \otimes n_i \tag{6}$$

$$I_1 = \mathrm{tr}B, I_2 = \frac{1}{2}\left[(\mathrm{tr}B)^2 - \mathrm{tr}B^2\right], \quad I_3 = J^2 = \det B \tag{7}$$

The deformation gradient F in the theory of finite strains can be decomposed into the product of the elastic deformation gradient F_e and the viscous deformation gradient F_v.

$$F = F_e \cdot F_v \tag{8}$$

The velocity gradient tensor L in the current configuration is defined as follows

$$L = L_e + L_v = \dot{F}F^{-1} = \dot{F}_e(F_e)^{-1} + F_e\dot{F}_v(F_v)^{-1}(F_e)^{-1} \tag{9}$$