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High-accuracy simulation of orbital dynamics: An object-oriented approach

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ABSTRACT

The use of Keplerian or Equinoctial parameters instead of Cartesian coordinates as state variables in object-oriented spacecraft models is introduced in this paper. The rigid body model of the standard MultiBody library is extended by adding transformation equations from Keplerian or Equinoctial parameters to Cartesian coordinates, and by setting the former as preferred states, instead of the latter. The remaining parts of the model are left untouched, thus ensuring maximum re-usability of the model itself. The results shown in the paper demonstrate the superior accuracy and speed of computation both in the case of a point-mass gravity field, and in the case of more accurate gravity field models.

1. Introduction

The safe and satisfactory operation of a satellite, in terms of its mission objectives, is strongly related to the performance level of its on-board attitude and orbit control systems, which provide the ability to maintain a desired orientation in space (or, e.g., carry out predefined attitude maneuvers) and track a desired, nominal orbit in spite of the presence of external disturbances. In addition, the recent trend towards missions based on constellations or formations of small satellites has led to the formulation of even more complex control problems, involving the relative motion (both in terms of attitude and position) of more vehicles at a time. This has resulted in an increasing need for efficient tools in every domain involved in spacecraft design, and particularly in the area of control-oriented modelling and simulation.

The Modelica Spacecraft Dynamics Library ([7,8,12]) is a set of models (based on the already existing and well-known Modelica MultiBody Library, see [10]) which is currently being developed with the aim of providing an advanced modelling and simulation tool capable of supporting control system analysis and design activities both for spacecraft attitude and orbit dynamics. The main motivation for the development of the library is given by the significant benefits that the adoption of a systematic approach to modelling and simulation, based on modern a-causal object-oriented languages such as Modelica, can give to the design process of such advanced control systems.

At the present stage, the library encompasses all the necessary utilities in order to ready a reliable and quick-to-use scenario for a generic space mission, providing a wide choice of most commonly used models for AOCS sensors, actuators and controls. The library's model reusability is such that, as new missions are conceived, the library can be used as a base upon which readily and easily build a simulator. This goal can be achieved simply by interconnecting the standard library objects, possibly with new components purposely designed to cope with specific mission requirements, regardless of space mission

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scenario in terms of either mission environment (e.g., Planet Earth, Mars, Solar system), spacecraft configuration or embarked on-board systems (e.g., sensors, actuators, control algorithms).

More precisely, the generic spacecraft simulator consists of an Extended World model and one or more Spacecraft models. The Extended World model is an extension of the standard MultiBody. World model, which provides all the functions needed for a complete representation of the space environment as seen by a spacecraft: gravitational and geomagnetic field models, atmospheric models, solar radiation models. Such an extension to the basic World model as originally provided in the MultiBody library plays a major role in the realistic simulation of the dynamics of a spacecraft as the linear and angular motion of a satellite are significantly influenced by its interaction with the space environment. The Spacecraft model, on the other hand, is a completely reconfigurable spacecraft including components to describe the actual spacecraft dynamics, the attitude/orbit control sensors and actuators and the relevant control laws. In this paper we are specifically concerned with the SpacecraftDynamics model; this component has been defined by extending the standard Body model, which describes a six-degrees-of-freedom rigid body. The main modifications reside in the selectable evaluation of the interactions between the spacecraft and the space environment and on the additional initialization options for the simulation via selection of a specific orbit for the spacecraft.

The main drawback associated with the adoption of the standard Body component as the core of the Spacecraft model is related to the intrinsic use this component makes of the Cartesian coordinates in the World reference frame for the state variables associated with the motion of the Body's center of mass. Indeed, for spacecraft work it is well known that significant benefits, both in terms of simulation accuracy and computational performance, can be obtained by using different choices of state variables, such as Keplerian and Equinoctial parameters (see, e.g., [13,9]).

Therefore, the aims of this paper, which extends preliminary results presented in [2,3] are the following:

- to demonstrate improvements in terms of simulation accuracy and efficiency which can be obtained by using Keplerian and Equinoctial parameters instead of Cartesian coordinates as state variables in the spacecraft model;
- to illustrate how Keplerian and Equinoctial parameters can be included in the existing MultiBody spacecraft model by exploiting the object-oriented features of the Modelica language and the symbolic manipulation capability of Modelica tools.

The paper is organised as follows: first an overview of the available choices for the state representation of satellite orbits is given in Section 2; subsequently, the use of Keplerian and Equinoctial orbital elements for the simulation of orbit dynamics will be described in Section 3, while the corresponding Modelica implementation will be outlined in Section 4. The results obtained by the implementation and application of the proposed approach to the simulation of low earth and geostationary orbits will be presented and discussed in Section 5, with reference to a point mass gravity field, and in Section 6, with reference to a more accurate model of the gravity field.

2. Satellite state representations

The state of the center of mass of a satellite in space needs six quantities to be defined. These quantities may take on many equivalent forms. Whatever the form, we call the collection of these quantities either a state vector (usually associated with position and velocity vectors) or a set of elements called orbital elements (typically used with scalar magnitude and angular representations of the orbit). Either set of quantities is referenced to a particular reference frame and completely specifies the two-body orbit from a complete set of initial conditions for solving an initial value problem class of differential equations.

In the following subsections, we will deal with spacecraft subject only to the gravitational attraction of the Earth considered as a point mass (*unperturbed Keplerian conditions*) and we will refer mainly to the Earth Centered Inertial reference axes (ECI), defined as follows: the origin of these axes is in the Earth's centre; the X-axis is parallel to the line of nodes; the Z-axis is parallel to the Earth's geographic north–south axis and pointing north. The Y-axis completes the right-handed orthogonal triad.

2.1. Position and velocity coordinates

In the ECI reference frame, the position and velocity vectors of a spacecraft influenced only by the gravitational attraction of the Earth considered with punctiform mass will be denoted as follows:

$$r = [x \quad y \quad z]^T, \tag{1}$$

$$v = [v_x \quad v_y \quad v_z]^T = \frac{\mathrm{d}r}{\mathrm{d}t}. \tag{2}$$

The acceleration of such a spacecraft satisfies the equation of two-body motion

$$\frac{d^2 r}{\mathrm{d}t^2} = -\mu \frac{r}{|r|^3} \tag{3}$$

where $\mu = GM_\oplus$ is the gravitational coefficient of the Earth. A particular solution of this second order vector differential equation is called an orbit that can be elliptic or parabolic or hyperbolic, depending on the initial values of the spacecraft position and velocity vectors $r(t_0)$ and $v(t_0)$. Only circular and elliptic trajectories are considered in this study.