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Fractional order models for system identification of thermal dynamics of buildings

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ABSTRACT

This study presents a class of fractional order models for system identification of thermal dynamics of buildings. Fractional order dynamics has been found to be inherent in the nature of heat transfer problems. It is thus instinctive to use fractional order models to describe the overall thermal dynamics of a building. Besides, fractional time series modeling is known by its long memory effect and capability of representing high-order complicated models in lower-order and compact forms. The reduction of model parameters can then relieve the computational overhead in the system identification procedure. This is of particular significance in model-based predictive control for building energy efficiency. In particular, a fractional order autoregressive model with exogenous input (FARX) is formulated and a corresponding parameter estimation using least squares technique is also provided. Furthermore, the FARX model is validated using simulation data from a detailed model built via IES<VE> software and compared with the prediction using traditional ARX model. It is found that the FARX model can reduce the computational time largely while retaining the prediction accuracy.

1. Introduction

1.1. Building energy simulation and modeling

The recognition of the large amount of energy consumption attributed to the building sector [1,2] has lead to voluminous studies for improving energy efficiency of buildings, and eventually facilitating the realization of sustainable and energy-efficient 'Smart-Cities' [3]. The potential of energy savings is found approximately up to 30% by intelligent automation [4]. One accomplishment towards this end is the introduction of model-based predictive control (MPC) on heating, ventilation, and air conditioning (HVAC) of buildings. By means of this, approximately 17–24% energy savings can be realized with comparison to current industry approach, i.e. rule-based control, according to experimental studies [5–7]. Comparable performances are also manifested in simulations for varied types, scales and scenarios of buildings, such as [8–14] to name but a few. Noteworthy is that the implementation of MPC relies on efficient and accurate prediction in the prescribed forecast horizon [15,16]. This calls for a low-order model while of high accuracy in predicting building energy dynamics, because the computational overhead of model complexity may lead to intractable MPC problems.

Thermal behavior modeling of buildings is of interest itself in building energy community for providing key building indicators such as energy demand and temperature. At present, there are over hundreds of building energy programs for whole-building energy simulation, such as TRNSYS [17], EnergyPlus [18] and IES<VE> [19] and so on. A comparison of them can be found in [20]. The models built using these programs are detailed and complicated, often referred to as white-box models, and hence unsuitable for MPC. In this regard, recent years have witnessed the efforts spared in selecting/developing alternative models and corresponding system identification for MPC [15,16,21]. The data-driven modeling seems to be well-suited, such as state space model, autoregressive-moving-average model with exogenous input (ARMAX) and its variants. Correspondingly, their parameters are often estimated using subspace system identification and least squares method respectively. Still in the framework of autoregressive modeling, multi-step instead of one-step ahead prediction error is minimized to improve the forecast accuracy in the prediction horizon of MPC, leading to the so-called MPC relevant identification [22,15,23]. Applications and improvements of these methods and others as well can be found in [24–27]. The aforementioned models are referred to as black-box models, because no prior physical

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Nomenclature

α_i	the ith fractional order on the output
α	arbitrary fractional order
$\boldsymbol{\phi}(k)$	vectorized known parameters in the least squares estimation
$\boldsymbol{\theta}$	vectorized unknown parameters to be identified
$\beta_{p,j}$	the jth fractional order on the pth input
$\mathcal{D}$	fractional derivative operator
$\Gamma(\cdot)$	Gamma function
$\hat{\boldsymbol{\theta}}$	estimated $\boldsymbol{\theta}$
τ	integration parameter
a_i, a'_i	coefficients corresponding to the output of the continuous and discrete fractional order model
$b_{p,j}$, $b'_{p,j}$	coefficients corresponding to the pth input of the continuous and discrete fractional order model
$f(t)$	function of t
h	sampling time interval
i, j	indexes
$J(\boldsymbol{\theta})$	objective function
k	index of time instant of the FARX
L	number of past values in approximating Grünwald-Letnikov derivative
l	non-negative integer
m	positive integer
N	number of samples used in identification
n_a	number of fractional orders on the output
n_b	number of fractional orders on the input
n_k	input–output delay
n_u	number of input
p	system input index
Q_f	free gain of the simulated building
Q_h	controlled gain of the simulated building
t	time
T_a	temperature of ambient air
T_z	temperature of zone air
u_p	system inputs, indexed by p
$U_{p,j}$	parameter accounting for the effect of past values of the pth input
y	system output
Y_i	parameter accounting for the effect of past outputs
ARFIMA	autoregressive fractionally integrated moving average model
ARMAX	autoregressive-moving-average model with exogenous input
ARX	autoregressive model with exogenous input
FARX	fractional order autoregressive model with exogenous input
FIT	indicator of the fitness of identified model
HVAC	heating, ventilation, and air conditioning
IES<VE>	Virtual Environment by Integrated Environmental Solutions Ltd.
MAE	mean absolute error
MaxAE	maximum absolute error
MISO	multi-input single-output
MPC	model-based predictive control
MSE	mean squared error

information of the building are required. However, in practice, experimental data are not always available, and hence the white-box model is needed to generate informative input/output data for developing the black-box models, rendering a co-simulation strategy necessary [28].

Simplification, i.e. model order reduction, appears compulsive for implementing MPC; wherein it is also crucial to retain key physics of the building energy system. This is why state space representations are embraced by the community, as they can be derived directly from the electrical analogies of buildings [29,30]. Partial physical information of the building can also be conveniently included in the modeling. On the other hand, ARMAX model is often criticized for lack of physical interpretation, although in cases it even has better performance [31]. In this regard, a physical-based ARMAX model has been pursued in [32] and according to an extensive measurement over 109 days, the prior physical information can boost the modeling accuracy. Notwithstanding, it is still arguable that the determination of model order and the the selection of state variables can be tricky and subjective in state-space representation [33].

In other words, in extracting a simple black-box model required for MPC, from the experimental data or from the corresponding white-box modeling, it is essential to realize model-order reduction, e.g. through physical description and eigen-analysis [33,34]. At the same time, it is also important to preserve the physical fundamentals. To further explore these two aspects and hence facilitate the use of MPC in saving building energy, we herein present a class of fractional order models for building energy systems. The fractional order models found in literature are able to describe the nature of heat transfer problems; besides, characteristics of fractional time series modeling are suitable for building energy systems, including its long memory effect and the capability of expressing the model using a smaller number of parameters. These features are detailed in the following from the standpoint of modeling building thermal dynamics.

1.2. Fractional-order thermo-dynamics and fractional time series modeling

Fractional calculus is a natural extension of calculus of integer order to arbitrary order, with a history as long as the traditional calculus [35]. However, it has only found wider application in engineering in the past several decades, including the fields closely related to the building energy systems, namely physics, systems and control [36–39].

One of its successful applications is in thermodynamics to describe heat transfer problems which obey diffusion phenomenon. For example, heat conduction through a wall or a sphere was shown analytically to be of a fractional order of 0.5, and a fractional order model was presented and validated using an experimental setup [40]; beam heat process was found much more precisely described also by using the fractional order models [41] where the identified transfer function matched that obtained from measurement data accurately for a wide range of frequencies. On a system level, it was shown that a fractional order model of only a few parameters is able to describe the responses of a large network composed of hundreds of resistor and capacitors [42], and such networks can be used as electrical analogies of thermal behavior of buildings [29,25]. Therefore, fractional-order dynamics appears inherent for building thermal dynamics from a physical perspective.

In time series analysis, the autoregressive integrated moving-average model [43] was generalized by permitting fractional differencing, resulting in the so-called autoregressive fractionally integrated moving average (ARFIMA) models [44,45]. This family of models was intended to properly account for the dependence between distant observations of series particularly arising in economics, and has been widely used in economic forecasting [46], e.g. for electricity price prediction [47]. Correspondingly, techniques were developed for parameter estimation of ARFIMA models [48]. Note that in the ARFIMA model, exogenous input is not considered.