Engineering Structures 152 (2017) 45–56

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Laminated element analysis to predict the shear strength of concrete beams under distributed and concentrated loads

Jong-Chan Kim, Kyoung-Kyu Choi *

School of Architecture, Soongsil Univ., 369 Sangdo-ro, Dongjak-gu, Seoul 06978, Republic of Korea

ARTICLE INFO

Article history:
Received 16 May 2017
Revised 4 September 2017
Accepted 5 September 2017

Keywords:
Concrete beams
Shear strength
Distributed load
Concentrated load
Compression zone
Laminated elements
Critical shear cracks

ABSTRACT

In the present study, an advanced analytical method was developed to evaluate the shear strength of slender concrete beams under distributed and concentrated transverse loads. The concrete shear capacity of the concrete beams is mainly provided by the compression zone of intact concrete since the tension zone of the concrete beams is severely damaged by flexural cracks before shear failure. The concrete shear capacity is evaluated based on concrete material failure criteria addressing combined compressive normal and shear stresses in the compression zone along the critical shear crack surface. For complete analysis, laminated elements are applied to take into account the stress variation in each location of the compression zone. The proposed analytical method can be used not only to predict the shear strength, but also the location and the angle of the critical shear crack. The shear strength is predicted using the proposed method, and the results are compared to those of an experiment, covering a wide range of design parameters. In addition, analytical studies were performed for analytical beam models subjected to distributed and concentrated loads. Based on the analysis results, the effect of various parameters including the loading type (distributed and concentrated transverse loads) on the shear strength and the critical shear crack was investigated.

1. Introduction

The essential conditions and causes of concrete shear failure have not been still completely understood and need to be studied. Thus, practitioners design concrete structures with conservativeness as a widely-held belief. However, this may result in designs that are not cost effective. Over decades, many experimental studies have investigated the behavioral characteristics of concrete members that fail in shear [31].

Test results of slender concrete beams have shown that extensive flexural cracks in the tension zone occur prior to shear failure [1–5]. Flexural cracks can grow into diagonal tension cracks that penetrate the compression zone, and the diagonal tension crack then causes a sudden loss in strength followed by beam failure [4,6,7]. Prior tests [8–11] have found that the shear strength of concrete members is mainly affected by the flexural reinforcement ratio and concrete compressive strength. According to Tureyen and Frosch [12] as well as Zararis and Papadakis [4], this is because a high flexural reinforcement ratio increases the depth of the compression zone, and a high concrete compressive strength increases the concrete tensile strength along the diagonal tension crack.

Most concrete shear tests have been performed under a concentrated transverse load partly because the concentrated load is expected to generate a critical loading condition (e.g., a high shear force and flexural moment) and partly because it has been difficult to apply a distributed transverse load to test specimens. The current design codes, including ACI 318-14 [13], were developed based on the test results using a concentrated load. However, Brown et al. [14] produced test results using twenty four reinforced concrete beams under two different loading types (concentrated and distributed loads) and found that the shear strength and crack patterns of the concrete beams were affected by the applied loading types. Tung and Tue [15] also performed tests on fourteen concrete beam specimens without web re-bars under two different loading types (concentrated and distributed loads) and three different boundary conditions (simply supported, cantilever, and continuous beams), and they observed that the shear strength of the concrete beams was affected by the distribution of the shear force and the flexural moment. In fact, since the real concrete members are normally subjected to combined distributed and concentrated transverse loads, the member force distribution of the real concrete members is different from that of simply supported beams

* Corresponding author.
E-mail address: kkchoi@ssu.ac.kr (K.-K. Choi).

http://dx.doi.org/10.1016/j.engstruct.2017.09.002

subjected to a concentrated transverse load, which have usually been simulated in laboratory tests.

In concrete shear design, the location of a critical shear crack is also an important issue to ensure structural safety, but its location is difficult to predict because it is influenced by various parameters, including the shear strength, force distribution, and concrete failure mechanism (Choi et al. [5,7]). Krefeld and Thurston [3] used extensive test results to suggest the location of a critical shear crack as a function of the shear span-to-depth ratio a/d. Zararis and Papadakis [4] suggested a theoretical location for the critical shear crack considering the geometrical conditions of the compression zone depth c and the angle of the critical shear crack. However, such models have been based on test results or theoretical investigation of simply supported concrete beams under a concentrated load and thus are not applicable for real concrete members that are subjected to different loading and boundary conditions. According to the test results by Zararis and Zararis [1], the location of the critical shear crack of concrete beams under a distributed load is closer to the supports than that under a concentrated load, as shown in Fig. 1 [16,17]. Thus, the effect of the distributed and concentrated transverse loads on the shear strength and critical shear crack location of the concrete beams needs to be investigated to understand the shear behavioral characteristics of the concrete structures.

An analytical method is developed here for concrete beams under distributed and concentrated transverse loads. The shear strength of concrete beams was evaluated based on an assumption adopted in preceding studies [5,7]: the concrete shear capacity of the beams is mainly provided through the compression zone of intact concrete, and the capacity can be evaluated based on criteria for concrete material failure. However, this study is different from preceding studies [5,7] in that it presents a sophisticated, versatile analytical method that is applicable to concrete beams subjected to various loading conditions and is able to predict the shear strength as well as the location and the angle of the critical shear crack. This method was developed by considering the compressive stress distribution along the critical shear crack of the compression zone. This method adopts finite laminated elements to accurately evaluate the shear strength and predict the location and angle of the critical shear crack. For verification, the strength predictions of the proposed method were compared with existing test results that were performed for distributed and concentrated transverse loads. Furthermore, parametric studies using the analytical method were performed to understand the variations in the shear strength, and the location and angle of the critical shear crack according to various influence parameters including loading types.

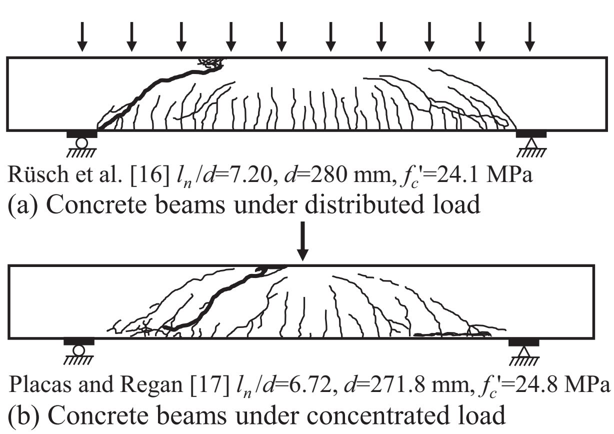

Fig. 1. Critical shear crack patterns of concrete beams under distributed and concentrated loads [16,17].

2. Shear strength evaluation of concrete beams

2.1. Shear capacity of concrete beams based on material failure criteria

In concrete slender beams, widespread flexural cracks develop ahead of shear failure and cause severe damage in the tension zone. One of the flexural cracks grows to a critical shear crack, which is a diagonal tension crack penetrating the entire compression zone, resulting in shear failure [1,3–5,14]. Thus, it is obvious that the concrete shear capacity is mainly provided by the intact concrete of the compression zone after significant flexural damage in the tension zone.

Choi et al. [5,7] theoretically defined the concrete shear capacity in the compression zone neglecting the shear contribution of the tension zone. In the model, the principal stresses were evaluated by using the compressive normal and shear stresses in the compression zone, and then Rankine's failure criteria was used to define the shear failure of the concrete compression zone. The preceding model was successfully verified by comparing the strength prediction with existing test results [5,17]. However, the preceding model mainly focused on the strength prediction of concrete beams under a concentrated load. Moreover, to develop a simplified design equation, the preceding model assumed an average compressive normal stress of the compression zone when evaluating the shear capacity instead of a direct consideration of the stress distribution developed by member forces. Thus, there is a limitation in applying the model to concrete beams that have been subjected to various loading and boundary conditions.

Fig. 2a shows an infinitesimal stress element located in the compression zone of concrete beams under a distributed transverse load. In the figure, an infinitesimal stress element is subjected to three stress components: compressive normal stress σ_{u1} developed by flexural moment in a member axial direction, compressive normal stress σ_{u2} in an orthogonal direction, and shear stress v_u. In Rankine's failure criteria, material failure could occur when the principal stresses (σ_1 or σ_2) reach the material strengths, that is, the concrete compressive strength (f_c') or concrete tensile strength (f_t), as shown in Fig. 2c.

For failure controlled by tension

$$\sigma_1 = -\frac{\sigma_{u1}+\sigma_{u2}}{2} + \sqrt{\left(\frac{\sigma_{u1}-\sigma_{u2}}{2}\right)^2 + v_u^2} \le f_t \tag{1a}$$

For failure controlled by compression

$$\sigma_2 = -\frac{\sigma_{u1}+\sigma_{u2}}{2} - \sqrt{\left(\frac{\sigma_{u1}-\sigma_{u2}}{2}\right)^2 + v_u^2} \le -f_c' \tag{1b}$$

where σ_1 and σ_2 are the principal tensile and compressive stresses. The tensile strength of concrete f_t is defined as $f_t = 0.21\sqrt{f_c'}$ (f_c' in MPa). σ_{u2} is developed by the distributed transverse load and was assumed to be w_u/b, where w_u is distributed load and b is beam width.

From Eq. (1), the shear stress capacity (v_{ut}) controlled by tension and that (v_{uc}) controlled by compression are determined as

For failure controlled by tension

$$v_{ut} = \sqrt{(f_t + \sigma_{u1})(f_t + \sigma_{u2})} \tag{2a}$$

For failure controlled by compression

$$v_{uc} = \sqrt{(f_c' - \sigma_{u1})(f_c' - \sigma_{u2})} \tag{2b}$$

Thus, theoretically the shear stress capacity can be defined as the minimum between v_{ut} and v_{uc}. However, since the shear stress capacity (v_{uc}) controlled by compression is generally greater than