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Non-linear in-plane buckling of shallow concrete arches subjected to combined mechanical and thermal loading

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ABSTRACT

In this paper, non-linear elastic pre-buckling and in-plane buckling analysis for a circular shallow concrete arch subjected to a uniformly distributed load and time-varying uniform temperature field is performed. Transient thermal strain and basic creep strain are considered, the latter modelled using a fractional derivative creep law, to investigate the coupling effects of time, temperature and geometric non-linearity on mechanical behaviour and stability boundaries. The first correspondence principle is invoked allowing the problem to be treated elastically and statically, with the non-linear equilibrium equations derived using the principle of virtual work. Numerical solutions to the variable order fractional derivatives are obtained through a finite-difference based discretisation scheme. Results show that the coupling effect between transient thermal strain and geometric non-linearity is significant as it influences pre-buckling behaviour and reduces buckling strength. Basic creep strain is less influential, causing a slight enhancement of the effects of transient thermal strain.

1. Introduction

As linear analyses of shallow arches lead to an over-estimation of anti-symmetric bifurcation and symmetric snap-through buckling loads, caused by geometric non-linearity, non-linear methods are required for their analysis. Due to the common application of shallow arches in civil engineering, coupling effects of this inherent geometric non-linearity and additional complexities, including temperature changes and viscoelasticity, require research attention.

The effects of uniform thermal loading on the behaviour and in-plane elastic stability of shallow steel arches were investigated [1–3]. Elevated temperatures induce compressive stresses in arches due to end restrained thermal expansion and cause upward deflections. These thermal induced stresses and displacements are enhanced in shallow arches due to geometric non-linearity. Additionally, uniform temperature loading may cause anti-symmetric bifurcation buckling or symmetric limit instability failure. When shallow steel arches are subjected to combined mechanical and uniform temperature loading, additional axial compressive forces are generated and the radial deflections are reduced [4–8]. Furthermore, the critical in-plane buckling loads increase with temperature. Pi and Bradford [9] studied the thermoelastic stability of shallow steel circular arches subjected to thermal gradients. The authors discovered that arches may experience snap-through or bifurcation buckling when subjected to a thermal gradient due to increasing bending and axial compression caused by curvature changes and axial expansion respectively. Furthermore, it was found that anti-symmetric buckling is the dominant case and symmetric buckling can only occur for slender shallow arches. Cai et al. [10] investigated the stability of shallow steel parabolic arches subjected to temperature gradients and mechanical loading. Results showed that critical buckling loads increased with an increasing uniform temperature field. Conversely, the critical loads decreased with an increasing temperature gradient. However, less work has been completed on the behaviour and stability of concrete arches at elevated temperatures. Bouras and Vrcelj [11] conducted a pre-buckling and stability analysis of shallow circular concrete arches subjected to combined mechanical and thermal loading in order to analyse the effect of transient thermal strain (TTS). It was found that when considering TTS, axial compressive force increased with temperature and the arch deflected downwards. Moreover, TTS significantly magnified the reduction in buckling strength of concrete arches at elevated temperatures. Basic creep strain was not considered.

Concrete and concrete-filled steel tubular (CFST) arches deform in time due the viscoelastic effects of creep and shrinkage. These quasi-static changes in the non-linear equilibrium configuration may induce loss of stability under the sustained load, despite

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initially being deemed as stable. The phenomenon of creep buckling has received research attention for both concrete [12] and CFST [13–16] arches. Axial force, bending moment and displacements increase with time due to creep and shrinkage. Subsequently, the creep strain is magnified due to the increased stress levels, causing the deformations and stresses to continually grow and reach an unstable equilibrium configuration at which buckling is possible [12]. The significance of this behaviour is magnified in shallow arches due to geometric non-linearities. As the creep behaviour of concrete is sensitive to temperature variations [17], typical daily and seasonal fluctuations should be considered in long-term analyses of concrete and CFST structures. Luo et al. [18] investigated the time-dependent behaviour of a crown-pinned circular CFST arch under constant temperature change and found that the coupling between creep, shrinkage and temperature significantly influenced the long-term deformations and internal forces. Wang et al. [19] studied the effect of temperature fluctuations on the creep behaviour of a CFST arch bridge. Using the micro-prestress solidification theory [20] and the age-adjusted effective modulus method, they found that creep deformations and stresses at the arch mid-span increased by 9% and 7–18% respectively, when temperature changes were considered. These findings emphasize the importance of temperature-time coupling in arch structures. However, these two studies were restricted to low temperature variations (not exceeding 100 °C) and not extended to analyse stability.

Despite not accurately reflecting the behaviour of real materials [21], viscoelastic models have been widely adopted for the constitutive relation of materials exhibiting time-dependent properties. The inaccuracy is due to the linear differential stress-strain equation being of integer order. To overcome this, viscoelastic chains are constructed which consist of a multitude of viscoelastic units placed in series or parallel. However, as complexities arise due to the myriads of material parameters to be characterised [22], intensive numerical simulations and comprehensive experimental data sets are required. Hence, the practicality of employing viscoelastic chains in both analytical and numerical investigations is diminished. The inherent disadvantages associated with viscoelastic models are overcome using fractional calculus, which is a branch of mathematical analysis concerned with performing integration or differentiation to a degree of real value [23–25]. Despite the formulation of fractional calculus in the 17th century, the most significant developments in engineering and scientific applications have been found only in the last 100 years [23]. With fractional calculus, the spring-dashpot models of viscoelasticity may now be generalised by replacing the integer order of derivative in the constitutive stress-strain equation with a real order. Fractional viscoelastic models, such as the fractional Kelvin-Voigt, fractional Maxwell and fractional Zener, have proven to be robust descriptors of material behaviour [26], as experimental data can be accurately reflected with the use of a minimal number of material parameters. Papoulia et al. [27] proved that the models of fractional viscoelasticity are obtained when the number of units in a generalised viscoelastic chain approach infinity. Additional applications of fractional viscoelasticity to model the dynamic behaviour of concrete includes the work of Barpi and Valente [28], who combined a micro-mechanical model and fractional viscoelastic element to investigate crack propagation in concrete, and Katicha and Flintsch [29] who employed fractional viscoelastic models to characterize the time-dependent properties of asphalt concrete. More recently, Bouras et al. [30] developed a fractional calculus based viscoelastic model for high temperature creep in concrete.

In the present study, an elastic in-plane buckling analysis of a shallow plain concrete arch subjected to a uniformly distributed radial load and time-varying uniform temperature field, for both pinned and fixed ends, is undertaken. The arch is assumed to be mechanically pre-loaded, so TTS will manifest when the concrete arch is heated. Basic creep strain is considered and modelled using the aforementioned variable order fractional derivative creep law. The influence of both transient thermal strain and basic creep strain on the behaviour and stability boundaries of shallow concrete arches under short-term temperature increases, are the focus of the investigation. Although creep strain is often neglected in studies of concrete under transient temperature increases such as those caused by fire, it warrants investigation in shallow arches due to their geometric non-linearities and increasing temperature-dependent stresses and deflections.

2. Material model

The total strain ϵ in pre-loaded concrete members subjected to an elevating temperature, a function of time t, stress $\sigma(t)$ and temperature $T(t)$, is defined as

$$\epsilon(\sigma,t,T(t)) = e(\sigma(t),T(t)) + e_{cr}(\sigma(t),T(t),t) + e_{th}(T(t)) + e_{tr}(\sigma(t),T(t)), \tag{1}$$

where e is the instantaneous mechanical strain, e_{cr} is the basic creep strain, e_{th} is the thermal strain and e_{tr} denotes TTS. Thermal strain is known as

$$e_{th} = a\Delta T, \tag{2}$$

with a representing the coefficient of thermal expansion, assumed constant at $8\times10^{-6}/^{\circ}\mathrm{C}$, and $\Delta T = T - T_0$, where T_0 is the initial temperature. Considering only elastic behaviour, the instantaneous mechanical strain can be expressed as

$$e_{el} = \frac{\sigma}{E(T)}, \tag{3}$$

where $E(T)$ is the temperature dependent elastic modulus which will be modelled using the Eurocode 2 formulation [31]. The Anderberg model of TTS [32] is adopted and defined as

$$e_{tr} = \frac{\sigma}{f'_c}\beta a\Delta T. \tag{4}$$

where f'_c denotes the cold compressive strength and the constant $\beta = 2.35$. This model has been employed as the TTS is linear proportional to stress which simplifies the analytical analysis, and due to its common application in fire investigations. Adopting other TTS models does not qualitatively change behaviour however may cause quantitative differences in results, see [11].

A rheological model consisting of a fractional dashpot (springpot) unit will be employed to model the basic creep strain which is governed by the following variable order fractional derivative equation, see [30],

$$D_t^{\alpha(T)} e_{cr} = \frac{\sigma}{\eta(T)}. \tag{5}$$

In Eq. (5), $D_t^{\alpha(T)}$ is the operator of the fractional derivative of variable order $\alpha(T)$ with respect to time t, satisfying $0 < \alpha < 1$ and $t > 0$, and $\eta(T)$ is the temperature dependent dynamic viscosity. These parameters are available in Appendix A.1. Two definitions of the variable-order fractional derivative are adopted and subsequently examined; the Caputo fractional derivative of a function $f(t)$ known as

$$D_t^{\alpha(t)} f(t) = \frac{1}{\Gamma(1-\alpha(t))}\int_0^t \frac{D^1 f(\tau)}{(t-\tau)^{\alpha(t)}}\, d\tau, \tag{6}$$

and the *memory of order* definition

$$D_t^{\alpha(t)} f(t) = \int_0^t \frac{D^1 f(\tau)}{\Gamma[1-\alpha(t-\tau)](t-\tau)^{\alpha(t-\tau)}}\, d\tau, \tag{7}$$