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Evolution of bridge frequencies and modes of vibration during truck passage

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ABSTRACT

This paper reports an experimental campaign that aims at measuring the evolution of bridge modal properties during the passage of a vehicle. It investigates not only frequency shifts due to various vehicle positions, but also changes in the shape of the modes of vibration. Two different bridges were instrumented and loaded by traversing trucks or trucks momentarily stationed on the bridge. The measurements were analysed by means of an output-only technique and a novel use of the continuous wavelet transform, which is presented here for the first time. The analysis reveals the presence of additional frequencies, significant shifts in frequencies and changes in the modes of vibration. These phenomena are theoretically investigated with the support of a simplified numerical model. This paper offers an interpretation of vehicle-bridge interaction of two particular case studies. The results clearly show that the modal properties of the vehicle and bridge do change with varying vehicle position.

1. Introduction

It is a well-known fact that the modal properties of two separate mechanical systems change when both systems interact. The coupled arrangement might have significantly different natural frequencies and modes of vibrations, compared to the uncoupled systems [1]. This is also acknowledged in bridge engineering to some extent, when investigating vehicles crossing the structure, i.e. it is understood that natural frequencies of a bridge change when heavy (massive) traffic traverses it.

As pointed out by Frýba [2] the fundamental frequency of a loaded beam depends not only on the magnitude of the mass on the deck but also on the position of the mass. A key factor in the scale of frequency variation that occurs for different mass positions is the ratio between the vehicle and bridge masses, with higher mass ratios producing larger shifts in the bridge frequency. Despite the general acceptance that such frequency shifts will occur, this is a problem not well studied in bridge engineering literature [3]. However, there have been some recent studies, for example [4] describes changes in the fundamental frequency of a railway bridge during passage of a train and provides an approximate formula to calculate changing bridge frequency. Yang et al. [3] study the variation of both vehicle and bridge frequencies and present a closed-form expression for a simply supported bridge considering only the first mode of vibration. Cantero & OBrien [5] investigate numerically the effect of different mass ratios and frequency ratios on the changes in system frequencies, where frequency ratio (FR) = vehicle frequency/bridge frequency and mass ratio (MR) = vehicle mass/bridge mass. The numerical analyses of coupled vehicle-bridge models in [5,6] show that for certain mass and frequency ratios it is possible to achieve positive frequency shifts in the fundamental frequency of the bridge. There exist only a limited number of studies that investigate this problem either experimentally, or in real operational bridges. For instance, in [7] the authors use a variety of output-only techniques with the response of a scaled model and are able to obtain clear frequency evolution diagrams for the case of large mass ratios. Also [6] performs a controlled laboratory experiment obtaining frequency shifts that validate an approximate closed-form solution of the frequency shift. The study in [8] investigates how a parked vehicle on an operational bridge affects its fundamental frequency, reporting frequency reductions of 5.4%. More recently, [9] explores the non-stationary nature of a 5-span bridge traversed by a truck, using alternative time–frequency tools, with limited success. Frequency is not the only modal property changing with load and its position; for instance [10] used numerical simulation to show that damping of a pedestrian bridge also changes according to number and location of pedestrians. That said, the majority of the limited papers available on the

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topic focus only on tracking frequency changes and do not evaluate the effect of load on the associated mode shapes.

Although a small number of authors have used numerical models to study the problem of frequency variation with load position, to date, no experimental investigation on full scale bridges has been presented. Such a study is the main contribution of this paper. Two separate experiments were carried out, each using a different test truck on different instrumented bridges. Bridge A is a three-span continuous structure monitored while a truck traverses it at a constant speed. The measurements from Bridge A provide only weak evidence of the evolution of the modal properties and hence it constitutes only a first attempt. A second experiment is reported on Bridge B, which is a single span bridge. For the experiment on Bridge B, a truck stops at certain locations on the bridge. The free vibration measurements of the bridge accelerations, right after the vehicle stops, allows for the precise extraction of the modal parameters of the coupled system. This is repeated for various vehicle stopping positions to obtain the variation of the modal properties with respect to vehicle position. It is important to note that the variation in modal properties reported here are specific to the two case studies investigated; since these variations strongly depend on the particular vehicle and bridge.

Over the course of the investigation, it is shown that a vehicle being present on the bridge results in a coupled system, such that modal analysis results cannot be interpreted as two separate systems (bridge and truck). The vehicle-bridge interaction is a non-stationary problem where the modal parameters change with vehicle location. In general, the ideas and results presented here are of interest to engineers and researchers involved in any vehicle-bridge interaction study. However, the findings reported here have particular consequences for the current research thread on extracting bridge modal properties from passing instrumented vehicles, e.g. [11–13]. In general, these publications acknowledge that there is vehicle-bridge coupling, but fail to consider the changes in modal properties with vehicle position. In these papers modal analysis techniques are often applied to the full length of the signal obtained during vehicle passage. However, attempting to analyse what is in effect a non-stationary signal with conventional modal analysis techniques developed for stationary signals will necessarily result in unreliable modal properties.

As well as demonstrating that the bridge acceleration signal recorded during the passage of a truck is non-stationary, this paper provides advice and insight on a number of related issues. First, a modified and novel approach for performing the Continuous Wavelet Transform (CWT) is presented, and is shown to be an effective signal processing technique to visualise variations in system frequencies. Next, the source of the additional frequency peak in the spectra of the forced (i.e. loaded) bridge acceleration signal is investigated. This is carried out using a relatively simple but insightful numerical model, and experimental data from Bridges A and B. Moreover, this paper shows for the first time that not only do the natural frequencies evolve during traffic passage, but that the shapes of the associated modes of vibration also evolve. For every vehicle location, the vehicle-bridge system features distinctly different modes. This is supported by a theoretical analysis of the problem, and carefully extracted experimental results. However, it should be noted that this paper only reports findings on the first longitudinal mode of the bridge, no torsional or higher modes are investigated.

The remainder of this paper has four primary sections. Section 2 provides a theoretical background on the numerical model, modal analysis, and signal processing techniques used in this study. Section 3 describes an experimental test where a truck was driven across a 3-span bridge. Additional frequencies were observed in the spectra of the recorded bridge response. A numerical model is used to postulate the origin of the additional frequency peak. However, to experimentally confirm the validity of the model predictions it was necessary to redo the experiment using a revised procedure where the truck would stop at a series of discrete locations on a bridge. The outcome of the revised experiment is reported in Section 4.

2. Methods

This section provides the reader a brief overview of the tools used throughout this study. Section 2.1 describes the numerical model that helps explain non-intuitive changes in modal properties observed in the experiments. Section 2.2 provides references on the modal analysis procedures employed to analyse the measured acceleration signals. Finally, Section 2.3 describes a modified form of wavelet analysis that is used to visualise variations in the system frequencies for the non-stationary acceleration signals recorded on site.

2.1. Numerical model

The coupled vehicle-bridge model was programmed in Matlab [14] and a pictorial representation of the numerical model is shown in Fig. 1. The truck is simulated as a sprung mass m supported on a spring k, where the spring represents the suspension of the vehicle. The bridge is simulated using a finite element beam model where each beam element has 4 degrees of freedom, namely a rotation and a vertical translation at each end of the element. Elemental matrices for this kind of element can be found in the literature, e.g. [15]. The beam is defined by its span L, section area A, modulus of elasticity E, second moment of area I and mass per unit length ρ. The location of the vehicle is defined by the distance from the left support (x) and in the simulations the vehicle can be positioned anywhere on the beam ($0 \leq x \leq \mathrm{L}$). The coupling between both systems, i.e. bridge and vehicle, can be written in terms of the beam element shape functions and the relative position of the vehicle within that element [16]. However, defining a sufficiently dense mesh that has a node exactly at the location of the vehicle reduces the complexity of the procedure. In that case the matrices of both systems are assembled diagonally, and the coupling terms are off-diagonal negative stiffness values that link together the appropriate degrees of freedom. As two different bridges will be modelled, (each with different boundary conditions), for now the boundary conditions of the model are indicated with question marks in Fig. 1. Models of this type have previously been presented in the literature [17].

Fundamentally, the purpose of this model is to allow the vehicle to be moved incrementally across the bridge and to track how the bridge frequency changes with the position of the vehicle. For a given vehicle position, the bridge frequencies and associated modes of vibration can be determined using an eigenvalue analysis. Simulating a multi-axle truck as a single degree of freedom sprung mass is a simplification, and for some applications it would be an over simplification. However, it is shown later that for the purpose of this study, where the primary interest is in explaining the evolution of frequency with respect to truck position, the model is effective. Initially values for area (A), second moment of area (I) and mass per unit length (ρ) were determined from the available bridge drawings. For the Young's Modulus (E), standard values for steel and concrete of $2 \cdot 10^{11}\ \mathrm{N/m^2}$ and $2 \cdot 10^{10}\ \mathrm{N/m^2}$ respectively were used. After getting an initial estimate of bridge frequencies from the model, the bridge properties (in the model) are revised so that the fundamental bridge frequency of the model matches the free vibration frequency observed on site, this is further described in Sections 3 and 4. For the vehicle, the spring stiffness (k) is adjusted so that the vehicle frequency in the model