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# A comparative study of numerical modelling techniques for the fracture of brittle materials with specific reference to glass

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ABSTRACT

This paper presents a comparative study on the available numerical approaches for modelling the fracturing of brittle materials. These modelling techniques encompass the finite element method (FEM), extended finite element method (XFEM), discrete element method (DEM) and combined finite-discrete element method (FEM/DEM). This study investigates their inherent weaknesses and strengths for modelling the fracture and fragmentation process. A comparative review is first carried out to illustrate their fundamental principles as well as the advantages for the modelling of cracks, followed by the state-of-the-art trial application in the example cases. An example of a glass beam subjected to low velocity hard body impact is examined as a plane stress problem. By evaluating the applicability of different models, the most desirable model for the entire dynamic fracture response is identified, and this is found to be the FEM/DEM. The FEM/DEM model is further examined by comparing results with the experimental data from high velocity and oblique impact tests. The study reveals that the FEM/DEM yields the most satisfactory results when modelling the dynamic fracture process of brittle materials such as glass.



## 1. Introduction

Fracturing commonly occurs in most brittle engineering materials, such as rock and glass. The typical characteristics of the material response to this type of failure are described as being linear and elastic until a fracture occurs without previously showing any obvious indication of yield or hardening. However, the typical features and original causes of the cracks are varied, and are often related to the nature of the material properties. For example, the heterogeneity and discontinuity of rock leads to variations in nucleation and coalescence in different directions. While glass is considered to be a homogeneous material, cracking is due to inherent pre-existing micro cracks or flaws. Rock failure is commonly related to long term static loading, for example causing slope instability [1,2], or the collapse of rock surrounding tunnels [3,4]. Glass failure, on the other hand, is more likely to be caused by dynamic loading such as bird strikes, or the impact of windborne debris [5–7]. In order to investigate the damage mechanisms in loading conditions beyond what can be tested in the laboratory, numerical methods are increasingly useful for modelling this type of brittle failure.

Modelling the fracture and fragmentation process of a brittle material has to resemble the behaviour of the material both prior to and after the breakage stage, and must capture the transition from continuity to discontinuity. The current numerical methods present significant difficulties for the modelling of this entire process. In order to explore the applicability of these methods, both the strengths and weaknesses of the available numerical methods are reviewed, and on this basis the numerical method which has the greatest potential for modelling the entire brittle failure process can be identified. The current numerical techniques commonly used are classified into continuum based methods and discontinuum based methods, depending on the underlying theory adopted.

Continuum based methods are represented by the finite element method (FEM) [8,9] and the extended finite element method (XFEM) [10,11]. FEM is probably the most widely adopted continuum method. However, the standard FEM experiences problems such as excessive mesh dependence when using the strain-softening model, and there are limitations in processing the continuum-discontinuum transition [12]. For instance, this method cannot capture the effect of size that is commonly

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observed in quasi brittle failure [13]. New techniques, such as a constitutive model based on continuum damage mechanics [14], a failed element elimination technique [15] and an adaptive FE technique [16] have been proposed to overcome those challenges. The extended finite element method (XFEM) is designed to model cracking in the FEM framework based on the partition of unity method (PUM) [11]. Special enrichment shape functions are employed to approximate the displacement field near the crack tips, which eliminates the need for remeshing following the crack growth, thus making the XFEM more flexible in dealing with the crack propagation problem.

Discontinuum based methods include the discrete element method (DEM) and the hybrid finite-discrete element method (FDEM). DEM was originally developed to describe the mechanical behaviour of assemblies of discs and spheres based on the use of an explicit numerical scheme [17], and has been extended to bulk materials and discontinuous solids. One significant advantage of this approach is its ability to model a crack as a true discontinuity using simple block contact and bond laws, dispensing with the need for complicated empirical constitutive law [18]. However, DEM has an inherent difficulty in achieving consistency between the identified values of the micro-parameters and the macroscopic behaviour, and inappropriate micro-parameters will result in unrealistic simulation results when comparing with the physical tests. Additionally, the determination of the neighbouring blocks, contact generation or elimination of blocks consumes a great deal of computation time and RAM [19].

FDEM was developed to combine the advantages of FEM and DEM, and aims to simulate the entire fragmentation process, including the transition phase from continuum to discontinuum for solids. One particular example of such a hybrid approach, known as the combined finite-discrete element method (FEM/DEM) [20], discretises the computational domain with triangular or tetrahedral elements, and inserts cohesive elements between adjacent element pairs. Crack growth is modelled by the failure of cohesive elements, so that crack nucleation can only develop along the edges of elements.

In addition to these approaches, several other numerical methods have also been applied to model the failure of brittle materials. For instance, cellular automatons (CA) [21] and hybrid DEM-SPH [22] are utilised for the simulation of multiple crack growth and geology processes. Smooth particle hydrodynamics (SPH) [6] and hybrid FEM-SPH [23] have been used to simulate the high velocity impact process of an object on a windscreen. Such methods are not included in this study as the relevant research is still limited and requires further investigation.

With so many approaches available, the applicability of these numerical methods should be investigated when applied to brittle failure problems. In order to investigate the consistency of the results obtained by different methods, and to demonstrate both their strengths and weaknesses when employed in simulating the fracturing process, this paper presents a comparison study on the numerical approaches with the most potential. A review of their underlying principles, as well as the advanced techniques for modelling cracks is first carried out, followed by their state-of-the-art application in fracturing modelling. A comparative study of modelling the dynamic fracture of a glass beam subjected to low velocity hard body impact under plane stress conditions is then conducted, and from this the most desirable method, i.e. FEM/DEM, is identified. Then the FEM/DEM models are further examined by comparing their results with two groups of experimental results for high velocity and oblique impact on glass. Through this study, a suitable method of simulating the fracture process of brittle materials such as glass is identified and validated.

## 2. Comparison of numerical techniques for fracture modelling under low velocity impact

In this section, a comparative study of modelling the dynamic fracture of a glass beam subjected to low velocity hard body impact under plane stress conditions is conducted using the four approaches (i.e. FEM, XFEM, DEM, FEM/DEM) that have been reviewed in Appendix A. The calculation using continuum based approaches (FEM, XFEM) is performed with ABAQUS [24], while the discontinuum based approaches are performed using PFC [25] (DEM) and Y2D (FEM/DEM), respectively. Fracture response or behaviour, including the crack pattern, impact force and energy evolution (e.g. strain and kinetic energy), is examined to compare the validity and applicability of each method.

### 2.1. Overview of simulation procedure

#### 2.1.1. Models employed for glass fracturing simulation

Glass is commonly seen as a homogeneous isotropic material with idealised linear elastic behaviour until a breakage, but it possesses excellent mechanical properties with a compressive strength of more than 700 MPa and a tensile strength of more than 30 MPa [26].

*2.1.1.1. FEM.* The cracking model traditionally developed for modelling concrete is adopted in FEM analysis for glass as its Mode I dominant brittle behaviour is similar to that of ceramic and concrete. The discontinuous cracking failure can be analysed in this smeared manner. The crack initiation is detected by a simple Rankine criterion when the maximum principal tensile stress exceeds the pre-defined tensile strength, and the subsequent crack propagation direction is normal to the direction of the maximum principal stress. The crack initiation is entirely based on a Mode I fracture, but its evolution considers both Mode I and II post failure behaviour by utilising the fracture energy based the post-cracking model (Fig. 1(a)) for Mode I, and the shear retention model (Fig. 1(b)) for Mode II.

The fracture-energy-based cracking model characterises the Mode I behaviour by the stress-displacement curve in which the displacement component normal to the crack surface can be defined by Eq. (1), which includes the energy required to cause a unit area of crack opening in Mode I ($G_f^I$) and the peak cracking stress $\sigma_{tu}^I$.

$$u_{no} = \frac{2G_f^I}{\sigma_{tu}^I} \tag{1}$$

The shear retention model characterises the Mode II behaviour by reducing the cracked shear modulus as the crack opens. The degradation shear retention factor $\rho(e_{nn}^{ck})$ is defined as a power function of the ratio of the opening strain $e_{nn}^{ck}$ in the maximum opening strain $e_{max}^{ck}$, as given by Eq. (2), then the cracked shear modulus can be obtained by multiplying the shear retention factor and the initial uncracked shear modulus.

$$\rho(e_{nn}^{ck}) = \left(1 - \frac{e_{nn}^{ck}}{e_{max}^{ck}}\right)^p \tag{2}$$

Then both crack initiation and propagation considering the Mode I and Mode II behaviour of glass can be modelled using the aforementioned approaches of FEM.

*2.1.1.2. XFEM.* Linear elastic fracture mechanics (LEFM) and phantom nodes are implemented into XFEM analysis in addition to its built-in settings (i.e. discontinuous jump function and asymptotic