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A form-finding method based on the geometrically exact rod model for bending-active structures

J. Bessini*, C. Lázaro, S. Monleón

Departamento de Mecánica de los Medios Continuos y Teoría de Estructuras, Universitat Politècnica de València, Camino de Vera s/n, 46022 Valencia, Spain

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ABSTRACT

In the field of bending-active structures, the complexity of finding beforehand the equilibrium configuration and the non-linearity of the structural response are main issues during the conceptual phase. The use of tools based on classical form-finding procedures as dynamic relaxation is the main trend today; different mechanical models with 3, 4 or 6 degrees of freedom have been implemented for modelling the bending effect. However, there is a well-established class of mechanical models which has been specifically designed to reproduce the behaviour of very flexible structures and has not been used so far in form-finding of bending-active structures. These are derived from the so-called geometrically exact (or Reissner-Simo) beam theory, and they are able to treat arbitrarily large rotations and displacements. In this paper, we present the development of a form-finding tool based on Reissner-Simo's theory and the dynamic relaxation method, in order to find the static equilibrium of the system. The choice of form-finding parameters as the target curve length and the kinematic constraints at beam ends will determine the shape of the final structure in the 'design-oriented' process. Several numerical examples on a range of structures are tested to validate the formulation.

1. Introduction

1.1. Motivation and background

Nowadays, modern architecture is focused on the search of efficient uses of technological and sustainable materials, high-tech concept-design-erection processes and the possibility to produce lightweight solutions with minimal impact and maximum elegance in shape. These ideas are reflected on bending-active structures, because they are assembled using slender members bent into attractive lightweight and shaped configurations. The term 'active-bending' refers to a category of structures in which bending is used in the process of shape configuration, obtaining structural schemes from elastic members such as beams or rods, which initially are straight and unstressed. However, the number of realizations is limited. The Mannheim Multihalle designed by Frei Otto, which is a pioneering modern application of the bending active principle [1]; the Ephemeral Cathedral of Creteil (Paris, France) by du Peloux et al., gridshell structure made of composite materials [2]; the temporary ICD/ITKE Research pavilion made of plywood at the University of Stuttgart and the permanent textile umbrella for Marrakech [3] are some of the built examples.

Due to the non-linearity of the structural response of the active members, the simulation of the process of deformation of bending-active elements is one of the main difficulties during the conceptual phase. Moreover, it is often not possible to obtain beforehand the equilibrium description of the system; its shape has to be determined as a result of a tight interaction between form, forces, material properties and boundary conditions [4]. Form-finding methodologies such as finite element models with non-linear virtual links that are shortened to reach the final form [5], and dynamic relaxation methods based on beam models with 3–6 DoFs have been proposed by several authors [6–14].

Despite the availability of different mechanical models to describe the kinematics of flexible members, most formulations covered in the literature [7–14] are based on Kirchhoff's theory [15], in which cross-sections remain almost normal to the centreline of the rod and almost planar, and seems adequate to handle the specificity of the mechanical problems posed by bending-active structures. However, among those mathematical models handling large rotations and displacements, the so called geometrically exact rod model or Reissner-Simo model has been specifically designed to reproduce the behaviour of very flexible structures.

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* Corresponding author.
E-mail address: juabesmu@mes.upv.es (J. Bessini).

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Reissner [16] and Simo [17] developed a non-linear theory for the deformation of flexible rods that can be considered as a special case of Cosserat's theory, in which the director vectors attached to material fibres in a cross-section are constrained to remain inextensible and orthogonal, but not necessarily normal to the deformed centreline tangents. This theory can be also interpreted as a non-linear version of Timoshenko's theory for shear deformable beams and offers the conceptual simplicity of the kinematic assumption together with the power of a direct theory for solving non-linear problems.

The assumptions of the geometrically exact model can be summarized as follows: cross-sections remain plane without changes of shape or size, although they may experience finite rotations as a rigid-body; warping effects are not modelled; the centreline can undergo extension, and shear deformations are allowed. Simo in 1991 [18], extended his previous work incorporating shear and torsion-warping deformation into the geometrically exact rod model. The consideration of warping would indeed allow to handle thin-walled open cross-sections and the authors hope to extend their work in this line. Hence, taking into account the potential of the geometrically exact rod model to handle arbitrarily large rotations and centreline displacements without limitations of magnitude, the authors present in this work a form-finding methodology for bending-active structures based on this mechanical model.

1.2. Recent trends in form finding methods for bending active structures

Traditionally, finite element models have been used for the load analysis and the simulation of the construction process of structures. In the case of evolutive processes and non-linear response, implicit resolution methods are often used to find equilibrium solutions. However, in the so called 'design-oriented' problems, where the main unknown is the geometrical configuration and the determination of deformations and stresses is not the main goal, the use of explicit methods has been reported by some authors [13,19] as more advantageous for those cases in which prescribed conditions are far from the equilibrium geometry. As a starting point in the conceptual phase, physical models provide good geometrical approximations (Fig. 1), however, the initial pre-stress induced by the form-change is unknown. In this case, the development of numerical tools based on explicit methods as dynamic relaxation is the main trend today.

Dynamic relaxation provides a straightforward way to obtain the static equilibrium of the structure from a given arbitrary and inaccurate initial configuration. Moreover, the possibility of using fictitious stiffness, massess and time steps contributes to the fast convergence of the numerical solution. Several approaches for this method have been suggested over the years, being reduced models with only three translational DoFs the most used. Douthe et al. [8], focusing on the previous work done by Adriaenssens and Barnes [7] proposed to calculate the magnitude of bending moments through estimates of curvatures obtained from the positions of groups of three consecutive nodes. Bending moments are transformed into equivalent forces, acting on the actual geometry of the beam. This model is valid for initially straight beams with axisymmetric cross-section. The problem of torsion and bending for dynamic relaxation modelling of spatial curved beams was approached by Barnes et al. [9] few years later, extending Douthes work; their assumption accounts for both torsion and transverse (out of plane) bending effects; this model is valid in cases where cross-sections are isotropic.

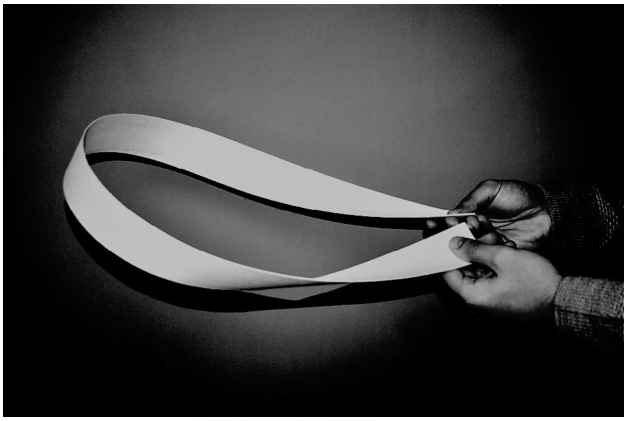

Fig. 1. A physical model subject to bending and torsion effects. (Bending of developable surfaces has been studied in [20].)

The first proposal to handle the problem of slender beams in large displacements using dynamic relaxation was introduced by Wakefield in 1980 [21]; he implemented a formulation taking into account 6 DoFs. As reported by Adriaenssens [22], Williams developed a more precise resolution scheme, assuming local reference frames at nodes and cubic shape functions to simulate the stiffness of the rod. Few years ago and based on Williams's work, D'Amico et al. [10] built a numerical framework aimed to the design of gridshells. Senatore and Piker [12] used a corotational approach to compute the resultant field of displacements and the effect of large deformations. The use of 6 DoFs per node has the advantage of providing a complete description of the mechanical response. However, Adriaenssens and Barnes mention in reference [7] that the coupling between translational and rotational DoFs can cause conditioning problems in explicit methods.

In recent years, the study of thin flexible rods has also been developed in the field of computer animation and games since this industry requires fast simulation techniques. For example, Bergou et al. [23] present a discrete treatment based on the study of adapted framed curves and the topological concepts of parallel transport and holonomy. In contrast to typical simulation techniques relying on a kinematic description, they treat the centreline as dynamic and the material frame as quasi-static defined by its angular deviation from the natural Bishop frames, that are torsion-free trihedrons related to Frenet's trihedrons by a rotation around the tangent vector. Thereby, solutions are formulated with a minimal number of degrees of freedom: the explicit description of the centreline of the rod (three DoFs) and the orientation of material frame using a scalar variable (one additional DoF). The authors claim that using this procedure, numerical instabilities are avoided in the dynamic relaxation algorithm. Based on these developments in the field of computer animation, du Peloux et al. [11], D'Amico et al. [13] and Lefevre et al. [14] have recently presented alternative approaches to model elastic rods with 4-DoFs. These formulations allow to deal with general cases of structures with anisotropic cross-sections and torsional stiffness effects by means of Bishop frames. In the beam model developed by D'Amico et al., the definition of boundary conditions is not straightforward and pre-processing is necessary, defining extra-nodes or constraints.

Different strategies have been carried out to perform dynamic relaxation in order to obtain accurate and fast simulations, having in common Kirchhoff's model as kinematic model. However, the authors propose the use of a mechanical model specifically designed to deal with large displacements and rotations. An additional advantage of the model is that it is appropriate for modelling structures in which shear deformation is not negligible. In the realm of bending-active structures, it could be applied to structures formed by two o more layers of laths, considering them as unitary structural elements with large shear deformations (e.g.