Engineering Structures 152 (2017) 750–757

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Enhancing the teaching of structural dynamics using additive manufacturing

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A R T I C L E I N F O

Article history:
Received 24 July 2017
Revised 16 August 2017
Accepted 25 September 2017

Keywords:
Structural dynamics
Flexural stiffness
Thermoplastic
3D printing
Education

A B S T R A C T

This paper provides a companion study to a previous paper by the same author (Virgin, 2017). In that paper, 3D printing was used to provide a hands-on experience for students of (linear) structural analysis based on the lateral stiffness of plane frames. In this paper, a related set of structural plane frames is investigated in terms of their natural frequencies, perhaps the fundamental feature in structural dynamics. Again, a 3D printer is used to provide a variety of parameter variations, and the extent to which certain simplifying analytical assumptions are justified is assessed.

1. Introduction

Rectangular, plane, portal frames offer compelling pedagogical opportunities in the context of learning structural dynamics. Their relatively simple geometric form allows a variety of comparative studies to be made, over and above single structural members. Additive manufacturing is an ideal mechanism for producing nominally similar structures to within relatively high specifications, and the widespread availability of 3D printers means that it is possible to incorporate an accessible practical aspect to otherwise theoretical studies in structural dynamics. Obtaining the natural frequencies of plane frames is a relatively simple task, especially for frames consisting of slender members, such that the behavior is dominated by elastic flexure [2]. Analysis is especially simple for frames undergoing sway motion - often the case for practical geometries associated with buildings in which floors are relatively stiff in comparison with columns, for example.

The analysis for free vibrations tends to be a little more involved than for stiffness, since mass effects need to be considered. Furthermore, in terms of modal analysis, the lowest few frequencies may be important even in linear regimes. Part of the reason for this is that under external excitation, any of the lowest modes can be excited and dominate subsequent dynamic behavior, especially in systems with relatively low damping. However, in terms of practical testing the extraction of frequencies is a standard procedure, once a time series has been measured. We shall see that the vibrational behavior of rectangular portal frames can be described in terms of various parameter ratios, essentially relating the distribution of stiffness and mass between the beam and columns, and providing an accessible means of allowing a schematic parametric study to be conducted.

Understanding the dynamic behavior of structures has also proved to be useful in damage assessment [3] and finite element model updating [4,5], and of course earthquake engineering is a natural context for this type of structural dynamics [6].

1.1. Some basic modeling

A spring (of stiffness coefficient K) attached to a mass (M), is modeled, via Newton's second law, by a second-order ordinary differential equation:

$$M\ddot{X} + KX = 0, \tag{1}$$

where the overdot means differentiation with respect to time, i.e., $\dot{X} \equiv dX/dt$, and given some non-trivial initial deflection $X(0)$ (away from equilibrium at $X = 0$) the mass will oscillate according to

$$X(t) = X(0)\cos\omega_n t \tag{2}$$

in which the key response characteristic is the natural frequency:

$$\omega_n = \sqrt{K/M}. \tag{3}$$

This harmonic motion reflects the continual exchange of energy between the potential energy stored in the spring ($V = \frac{1}{2}KX^2$) and kinetic energy associated with the moving mass ($T = \frac{1}{2}M\dot{X}^2$).

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https://doi.org/10.1016/j.engstruct.2017.09.052
0141-0296/

This spring-mass system and its natural frequency are representative of a wider class of structural system in which the stiffness and mass properties are typically associated with elastic deformation and distributed mass. Thus, simplistically, we see that the stiffer a structure, the higher the frequency of vibration, and the greater the mass, the lower the natural frequency. All mechanical and structural systems possess some form of energy dissipation or damping, thus oscillations have a tendency to decay in the absence of external excitation. However, in similarity with many practical systems (including the experimental systems to be described later) this damping will be relatively small, and will not have any significant effect on natural frequencies.

Given a rectangular, planar, portal frame, it is instructive to assess how natural frequencies depend on a range of parameters including material properties, boundary conditions, and geometry. It is the latter that will provide the primary focus of interest in this paper. For this type of continuous system there are an infinite number of natural frequencies, but it is often the case that they can be modeled by a very much lower-order model, and sometimes using a single-degree-of-freedom (SDOF) model. Consider a square portal frame, in which each of the three members is made of the same material and has the same length, as shown in Fig. 1(a). However, the cross-beam is much stiffer than the columns. The bottom of each vertical member is fixed to a rigid base and the top ends join the cross-beam with moment-transmitting joints. This system can be modeled as a SDOF system with the lateral deflection X completely describing the overall deflection of the system: *pure sidesway*.

The simplified model can be developed using the well-known concept of an equivalent (lumped) spring and mass. Details of this approach can be found in a number of texts, for example [7], but essentially use is made of equivalent potential and kinetic energies. The deflected shape of the frame under consideration depends entirely on the columns: there are many functions that can describe their shape (independently of magnitude). For example, the following polynomial for lateral deflection w satisfies the boundary conditions of zero rotation at both ends of the structural member ($w' = 0$ at $x = 0$ and L) but allowing one end to translate (sway) relative to the other [8]:

$$w(x) = \left(1 - \frac{2x}{L}\right)^3 - 3\left(1 - \frac{2x}{L}\right) + 2. \tag{4}$$

It is also well-established that for a beam-like structural component the strain energy stored in flexure is $V = \frac{1}{2}EI\int_0^L (w'')^2 dx$, where $w = w(x)$ is the deflected shape, and EI is the flexural rigidity, i.e., the product of Young's modulus E and the second moment of area I. Evaluating this expression using Eq. (4), and equating it with the potential energy of a spring, we obtain an equivalent stiffness $K_e = 12EI/L^3$ (for a single column).

Similarly, the effective mass associated with the sway motion of a beam (column) can be obtained using the kinetic energy for the continuous system $T = \frac{1}{2}\rho A\int_0^L \dot{w}^2 dx = \frac{1}{2}m\int_0^L \dot{w}^2 dx$, where ρ is the density and m is the mass, both per unit length. We can then equate this with the point-mass kinetic energy $T = \frac{1}{2}M_e v^2 = \frac{1}{2}M_e\dot{X}^2$, and again using Eq. (4) as the shape function we obtain $M_e = 0.37$ mL. We shall see later that these values are associated with the (1,1) elements in the beam stiffness and consistent mass matrices. It is not surprising that only a portion of the total mass contributes, given the fact that one end is not moving.

Thus, for a column vibrating with pure-sway motion as shown in Fig. 1(a), we have a natural frequency $\omega_n = \sqrt{K_e/M_e} = \sqrt{12EI/0.37\rho AL^4} = 5.69\sqrt{EI/mL^4}$, where $m = \rho A$ is the mass per unit length. The coefficient is much lower than that for the (much stiffer) clamped-clamped beam with no sway (22.37), which is associated with a symmetric vibration mode [9]. Solving the governing partial differential equation [8], results in a lowest natural frequency $\omega_1 = 5.59\sqrt{EI/mL^4}$; the slight difference with the lumped model results from Eq. (4) not exactly describing the shape of the column during the motion.

In the context of a rectangular frame in pure sway, we thus have columns that supply the sway stiffness and a rigid bar that supplies the mass (in addition to a smaller amount of effective mass contributed by the columns). For a rectangular portal frame we thus have [8]

$$f_1 = \frac{1}{2\pi}\left[\frac{12\Sigma((EI)_c)}{L^3(M + 0.37\Sigma M_c)}\right]^{1/2}, \tag{5}$$

in which M is the total mass of the beam, and the subscript c refers to the column. For frames made of the same material throughout the Young's modulus E and density ρ provide a simple square root scaling, $f \propto \sqrt{E/\rho}$. We see that unless the mass of the columns is negligible (relative to the mass of the beam) the natural frequency depends in a non-simple way on the properties of the columns. But, suppose the mass of the columns is very small compared to the mass of the beam: $M \gg \Sigma M_c$. Then we can more easily isolate the effect of varying a single parameter. For example, with all other things held equal, we have $f \propto (L)^{-3/2}$ for a changing column length. We also see that the natural frequency is independent of the width b (since both the stiffness and mass scale linearly with b and thus cancel). This is a slightly counter-intuitive result but also applies

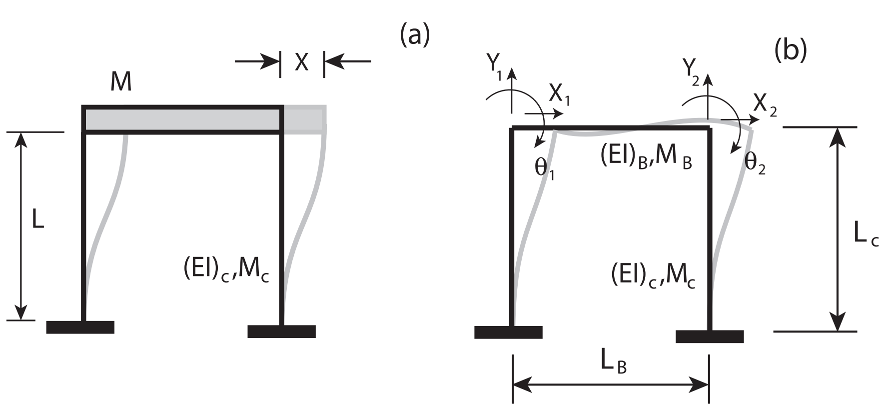

Fig. 1. Rectangular (portal) plane frames. (a) the pure sway case, (b) the more general case.