Engineering Structures 151 (2017) 235–252

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A maximum filter for the ground structure method: An optimization tool to harness multiple structural designs

Emily D. Sanders [a], Adeildo S. Ramos Jr. [b], Glaucio H. Paulino [a,*]

[a] School of Civil and Environmental Engineering, Georgia Institute of Technology, 790 Atlantic Drive, Atlanta, GA 30332, USA
[b] Federal University of Alagoas, Maceió, AL, Brazil

ARTICLE INFO

Article history:
Received 30 January 2017
Revised 21 June 2017
Accepted 20 July 2017

Keywords:
Topology optimization of trusses
Filtering
Tikhonov regularization
Nested elastic formulation
Plastic formulation
Max filter
Reduced order model

ABSTRACT

The ground structure method seeks to approximate Michell optimal solutions for real-world design problems requiring truss solutions. The single solution extracted from the ground structure is typically too complex to realize directly in practice and is instead used to inform designer intuition about how the structure behaves. Additionally, a post-processing step required to filter out unnecessary truss members in the final design often leads to structures that no longer satisfy global equilibrium. Here, a maximum filter is proposed that, in addition to guaranteeing structures that satisfy global equilibrium, leads to several design perspectives for a single problem and allows for increased user control over the complexity of the final design. Rather than applying a static filter in each optimization iteration, the maximum filter employs an interval reducing method (e.g., bisection)to find the maximum allowable filter value that can be imposed in a given optimization iteration such that the design space is reduced while preserving global equilibrium and limiting local increases in the objective function. Minimization of potential energy with Tikhonov regularization is adopted to solve the singular system of equilibrium equations resulting from the filtered designs. In addition to reducing the order of the state problem, the maximum filter reduces the order of the optimization problem to increase computational efficiency. Numerical examples are presented to demonstrate the capabilities of the maximum filter, including a problem with multiple load cases, and its use as an end-filter in the traditional plastic and nested elastic approaches of the ground structure method.

1. Introduction

Since Michell's 1904 landmark paper [1], in which he proposed criteria for minimum volume structures that equilibrate a set of forces (see also [2]), much work has been devoted to designing structures at "the limits of economy." For instance, optimal frames satisfying Michell's criteria have been analytically derived for various beam structures by e.g., A.Chan [3] in the 1960s, H. Chan [4] in the 1970s, and Lewiński, Zhou, and Rozvany [5,6] in the 1990s. Since analytical solutions are difficult or impossible to obtain for some practical design problems, others have turned to numerical approximations to Michell solutions. For example, the ground structure method, developed by Dorn et al. in 1964 [7], begins with a dense truss network composed of a finite number of members and uses numerical optimization to size the members and obtain approximate Michell trusses. More recent implementations demonstrate the efficiency of the plastic formulation of the ground structure method for finding approximate minimum volume trusses with bounded member stresses [8–12]. Although the plastic formulation is extremely efficient (it can be posed as a linear programming problem), it has limitations in extending to more complex problems [13]. Thus, this work focuses on the elastic formulation for volume constrained compliance minimization, which has been shown to be equivalent to compliance constrained volume minimization up to a scaling [14] for linear problems.

Both the plastic and elastic formulations of the ground structure method typically lead to highly complex geometries that are impractical in practice. Some work has been done to tailor the methods to obtain more practical designs. For example, Tugilimana et al. [15] introduced the concept of modularity into the formulation to obtain trusses consisting of multiple identical pieces that can be prefabricated offsite. Prager [16], and more recently Asadpoure et al. [17], introduced penalty terms in the objective function to reduce the number or weight of connections in their designs. Ramos Jr. and Paulino [18] recently introduced the so called discrete filter that changes the ground structure method for maximum stiffness design from a truss sizing optimization problem to a true topology

* Corresponding author.
E-mail address: paulino@gatech.edu (G.H. Paulino).

http://dx.doi.org/10.1016/j.engstruct.2017.07.064
0141-0296/

optimization problem in which a zero lower bound can be imposed and cleaner final designs can be obtained. The intermediate structures generated during the optimization are filtered by removing "unnecessary" members, while preserving global equilibrium and limiting local increases in the objective.

This work presents a maximum filter, which leads to several design perspectives for a single problem and allows user control over the final design. Accordingly, the contributions of the present work are as follows:

1. *Adaptive filter*: In contrast to the previous static filter [18], the max filter is adaptive. The magnitude of the max filter varies during the iterations in accordance with a user prescribed tolerance on the change in the objective, allowing for a broader range of possible designs and easier control over the final topology. Moreover, there is no need to set a specific value of the filter "a priori."
2. *Piecewise convexity*: Rather than applying the filter in every iteration, "piecewise convexity" is achieved by controlling *when* the max filter is applied, again leading to easier control over the final design.
3. *Efficiency*: In addition to reducing the size of the state equations, the size of the optimization problem is reduced when the max filter is applied, further addressing a major drawback of the nested elastic approach: computational cost.
4. *End-filter*: The max filter is shown to work effectively as an *end*-filter to guarantee designs obtained using the traditional plastic and nested elastic formulations satisfy global equilibrium.

Equipped with these features, the max filter becomes an effective engineering tool that can provide multiple perspectives on a given design problem and empower engineers and architects to take creative risks.

The remainder of this paper is organized as follows: Section 2 discusses the nested elastic formulation in a general sense. Section 3 provides the standard nested elastic formulation and the modified version used in this work. In Section 4, the max filtering scheme is detailed for use with the modified nested elastic formulation and for use as an end-filter with the traditional plastic and nested elastic formulations. Section 5 provides a brief review of solving the singular system of equilibrium equations using minimization of potential energy with Tikhonov regularization (PE-TR), and demonstrates the benefits of the method with a simple example. The use of a reduced order model (ROM) on both the state problem and the optimization problem as well as the implications on computational efficiency are discussed in Section 6. Aligned nodes and hanging members are addressed in Section 7, and some numerical aspects of the implementation are addressed in Section 8. In Section 9, three numerical examples are used to demonstrate the capabilities of the max filter, ROM, and the applicability of the max filter as an end-filter in the traditional plastic and nested elastic formulations of the ground structure method. Conclusions are presented in Section 11. Nomenclature used throughout the paper can be found in Appendix A, some comments on fully stressed designs in Appendix B, the max filter algorithm flowchart in Appendix C, a derivation of compliance for discrete optimal trusses discussed in the text in Appendix D, and details on solving singular systems in Appendix E. The MATLAB implementation is included as electronic supplementary material and a tutorial for using the code is provided in Appendix F.

2. On nested elastic formulations

It is widely known that the nested elastic formulation typically requires a small positive lower bound on the member cross-sectional areas to ensure that the problem remains well posed. As a result, the nested elastic formulation of the ground structure method becomes a truss-sizing problem in which all members defined in the initial ground structure are present in the optimal structure. Thus, the optimal solution contains many thin members. Discrete designs from ground structures are typically obtained by using the small positive lower bound on the design variables (or another arbitrary threshold value) as a "post-processing filter" that removes a given level of thin members once the sizing problem is complete [14]. This method of obtaining the final topology at the end of the sizing problem using an arbitrary threshold will be referred to as a "cutoff" in the remainder of this manuscript.

A number of issues arise when using the nested elastic formulation of the ground structure method for maximum stiffness design with a lower bound on the design variables and a cutoff. First, the final topology can depend largely on the value of the small positive lower bound and cutoff. These values must be carefully selected: the lower bound must be small enough to prevent non-optimal members from contributing stiffness, but large enough that the stiffness matrix does not become ill-conditioned. The cutoff should be small enough that critical structural elements are not removed from the final topology, but not so small that thin members remain in the final topology [19,14]. Second, when using this approach, truss members are often removed using the cutoff after the sizing problem is complete without regard for whether the final topology satisfies global equilibrium. In fact, solutions based on this approach often contain hanging members (i.e., members that are not connected to the structure at one or both ends) or internal mechanisms. Lastly, the value of the objective function is based on the result of the sizing problem in which all truss members from the initial ground structure are present, whereas the final topology after applying the cutoff actually represents an increased objective.

Fig. 1 shows three final topologies obtained for an 8×4 rectangular domain, clamped at one end, with a mid-height point load at the other end (Fig. 1a). The results are based on the nested elastic formulation with a small positive lower bound ($x_i^{min} = 1.581 \times 10^{-12}$) and various cutoffs. All three designs are obtained from a full-level ground structure based on a 9×5 nodal

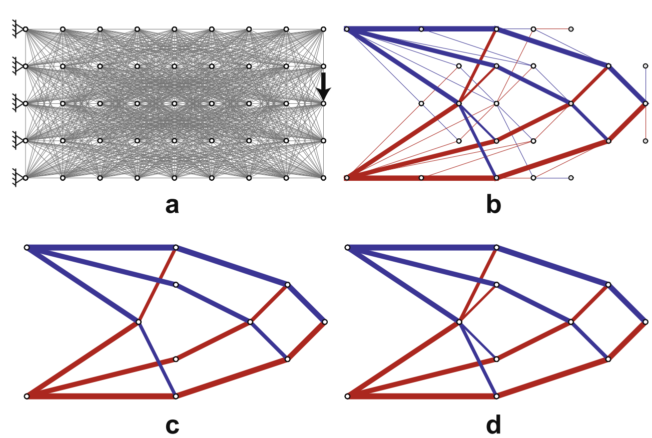

Fig. 1. Cantilever beam, clamped at one end with a mid-height point load at the other end - the optimization is based on a 9×5 nodal mesh and a full-level initial ground structure (632 non-overlapping members): (a) initial ground structure and boundary conditions; (b) final topology based on a 1.0×10^{-7} cutoff, which contains undesirable thin and hanging members; (c) final topology based on a 0.20 cutoff, which leads to a mechanism; (d) final topology based on a 0.010 cutoff, which is statically determinate. Note: Red indicates compression and blue indicates tension based on the stress state at the end of the sizing problem. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)