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Closed form stability solution of simply supported anisotropic laminated composite plates under axial compression compared with experiments

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ABSTRACT

Closed form expression for the buckling load of generally anisotropic laminated composite simply supported thin plates is derived. The Rayleigh-Ritz displacement field approximation based on the energy approach introduced an upper bound solution compared to the FE results. Therefore, the critical stability matrix is used to obtain an accurate buckling formula. The effective axial, coupling and flexural stiffness coefficients of the anisotropic layup is determined from the generalized constitutive relationship using dimensional reduction by static condensation of the 6×6 composite stiffness matrix. The resulting explicit formula has an additional term, which is a function of the effective coupling and axial stiffness. This formula reduces down to Euler buckling formula once the effective coupling stiffness term vanishes for isotropic and certain classes of laminated composites. The closed form results are verified against finite element Eigenvalue solutions for a wide range of anisotropic laminated layups yielding high accuracy. Comparisons with a limited number of experiments are also performed showing good correspondence. A brief parametric study is then conducted to examine the effect of ply orientations and material properties including hybrid carbon/glass fiber composites. Relevance of the numerical and closed form results is discussed for all these cases.

1. Introduction

The use of laminated composites in aerospace, automotive, marine and civil engineering applications is ever growing due to their distinguished properties (high stiffness-to-weight ratio, high strength-to-weight ratio, fatigue and corrosion resistance). This growth has resulted in increasing the demand for better understanding the mechanics of laminated composites. Buckling behavior of composite members, plates and shells has been recently studied including [1–13]. Composite plates, like any traditional members subjected to axial compression, undergo stability issues prior to failure. However, not many research studies have focused on the buckling of composite plates. Silva et al. [1] developed a formulation of a generalized beam theory (GBT) to study local and global buckling behavior of fiber reinforced polymer composite open section thin-walled columns. The solution for buckling using GBT included solving the following eigenvalue problem:

$$(K + \gamma G)d = 0 \tag{1}$$

where K is the global linear stiffness matrix; G is the geometric stiffness matrix; and d is the eigenvector.

Silvestre and Camotim [2] developed a second order generalized beam theory (GBT) to predict buckling behavior for thin walled arbitrary orthotropic thin-walled members. The second-order GBT formulation was also compared with Bauld and Tzeng theory [3]. The results showed that the critical load exists for all isotropic or cross-ply orthotropic members. On the other hand, non-linear primary path is exhibited and no specific bifurcation is detected for asymmetric orthotropic layups. Rasheed and Yousif [4] used the energy approach to develop a closed form solution to predict buckling of thin laminated orthotropic composite rings/long cylinders under external pressure:

$$P_{cr} = 3\left(\frac{A_{orth}D_{orth} - B_{orth}^2}{A_{orth}R^3 + 2B_{orth}R^2 + D_{orth}R}\right) \tag{2}$$

where $A_{orth}, B_{orth},$ and D_{orth} constants are the extensional, coupling, and bending stiffness coefficients obtained from dimensional reduction of orthotropic behavior. The developed formula yielded improved results compared to some design codes. Rasheed and Yousif [5] developed a closed form solution for buckling of anisotropic laminated composite rings and long cylinders subjected to

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external hydrostatic pressure. They confirmed their analytical results against finite element solutions and concluded that the buckling modes are symmetric with respect to rotated axes of the twisted section of the pre-buckling solution in case of anisotropy. Xu et al. [6] developed an approximate analytical solution to predict buckling of a tri-axial woven fabric composite structure under bi-axial loading based on the equivalent anisotropic plate method. They concluded that the analytical solution gives an upper bound solution for buckling load and it can be used to predict buckling behavior for real world problems subjected to bi-axial loading. Using first order shear deformation and von-Karman type nonlinearity, Shukla et al. [7] estimated critical buckling loads for laminated composite plates with various boundary conditions under in-plane uniaxial and biaxial loading. Span to thickness ratio, plate aspect ratio, lamination scheme, number of layers and modulus ratio effects were considered in estimating the buckling behavior. Sun and Harik [8] developed analytical buckling solution of stiffened antisymmetric laminated composite plates with bending-extension coupling using analytical strip method (ASM) which was first developed by Harik and Salamoun [9] to analyze bending of thin orthotropic and stiffened rectangular plates. The results showed that plates with free boundary conditions contribute the weakest stiffening effect. Moreover, the number of layers of ply orientations equal to 0 and 90 had no effect on the critical buckling load since the coupling stiffness matrix vanishes.

Debski et al. [10] studied buckling and post-buckling behavior of thin-walled composite channel column sections experimentally. The results were compared with numerical solutions obtained from finite element models (Abaqus and ANSYS) and analytical-numerical method (ANM). Based on multi term Kantorovich method developed by Kerr [11], Shufrin et al. [12] developed a semi-analytical solution for buckling of symmetrically laminated rectangular plates with general boundary conditions under combined tension, compression, and shear. The results showed that the stability of angle-ply laminated plates improved under biaxial compression/tension and shear. Moreover, additional in-plane forces were created due to the in-plane restrains. Thai and Kim [13] proposed a closed form solution for buckling of orthotropic plates with two opposite simply supported edges using two variable refined plate theories. State space concept was used on Levy type solution to solve the governing equations. The results showed more accurate solutions than the higher order shear deformation theory. Heidari-Rarani et al. [14] studied the effect of angle-ply and cross-ply layups on the stability of E-glass/epoxy square composite laminated plates under axial compression with SFSF (S: simply supported, F: Free) boundary conditions analytically, numerically, and experimentally. Using Rayleigh-Ritz method, a semi analytical solution was developed and verified against the numerical analysis yielding accurate results. Using hand layup method E-glass/epoxy plates of four layers were made with angle-ply ($[\mp 30]_s$ and $[\mp 45]_s$) and cross-ply ($[0/90]_s$) stacking sequences. However, the semi-analytical and numerical results were overestimated compared to the experimental ones. Lopatin and Morozov [15] presented analytical formula to predict the buckling of composite cantilever circular cylindrical shell under uniform external lateral pressure using the generalized Galerkin approach. The proposed analytical formula was verified against finite element analysis using COSMOS/M software with SHELL4L element yielding an accurate estimate of the buckling load values. Using exact stiffness analysis and smearing method, Damghani et al. [16] investigated the global buckling of simply supported composite plates containing rectangular delamination. It was noticed that as delamination length and width increased, the buckling load decreased for a single rectangular delamination. Furthermore, the buckling load values decreased as the delamination moved toward the edge of the plate. Alesadi et al. [17] investigated free vibration as well as buckling of cross-ply laminated composite plates using the Isogeometric approach (IGA) and Carrera's Unified Formulation (CUF).

In this work, a generalized closed form expression for buckling of simply supported laminated composite plates subjected to axial compression is developed. The standard Rayleigh–Ritz approximation approach provided an upper bound for the critical buckling load. A new critical buckling formula was derived using the critical stability matrix while implementing the critical buckling mode of isotropic plates. Axial, coupling and flexural rigidities in 1D are determined using dimensional reduction by the static condensation approach starting with the 3D stiffness matrix. Moreover, finite element models for the plates are established using the commercial software Abaqus. Furthermore, glass fiber-epoxy plates were made and tested in the laboratory. The finite element numerical solution was compared to the closed form solution resulting in excellent agreement. The experimental results also showed good comparison with the newly developed closed form expression. A good agreement between all three types of results was observed, regardless of the complexity of the composite layups used.

2. Analytical formulation

A generalized closed form buckling formula for simply supported anisotropic laminated composite plates under axial compression is derived using Rayleigh-Ritz approximation.

2.1. Assumptions

1. Buckling takes place in the x-y plane about the weak axis (z-axis).
2. The y-axis runs through the thickness of the plate where the composite lamination takes place, Fig. 1.
3. The lamination angle (α) is measured with respect to the x-axis (i.e. 0° fibers run parallel to the x-axis and 90° fibers run parallel to the z-axis). Accordingly, the angle (α) is rotated about the y-axis.
4. Plane sections before bending remain plane after bending and perpendicular to the mid surface (i.e. simple beam theory holds).

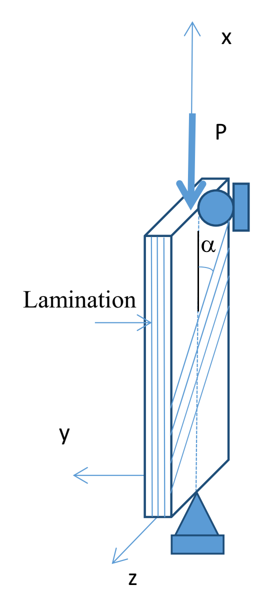

Fig. 1. The plate geometry.