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Some aspects of modeling and identification of inhomogeneous residual stress

R. Nedin *, V. Dudarev, A. Vatulyan

Institute of Mathematics, Mechanics and Computer Sciences named after I.I. Vorovich, Southern Federal University, 8a Mil'chakova Str., 344090 Rostov-on-Don, Russia
Southern Mathematical Institute, Vladikavkaz Scientific Center of Russian Academy of Sciences, 22 Markusa Str., 362027 Vladikavkaz, Russia

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ABSTRACT

Investigations of problems on mechanics of deformable solid body under residual (or initial, or pre-) stress state play a very important part in strength-and-stability assessment and in reconstruction of inhomogeneous properties. The present paper describes different theoretical aspects of modeling of residual stresses (RS). We compare several RS models for the vibrational problems for beams and plates and estimate numerically RS effect on their dynamical characteristics: frequency-response functions and eigenfrequency spectrum. We formulate and investigate the inverse problem on an identification of inhomogeneous plane RS state in a thin plate on the basis of the acoustic method. To solve the inverse problem, we propose two different approaches. We discuss the results of computational experiments on RS identification in detail and provide some practical recommendations on a selection of frequency range in order to obtain the highest reconstruction accuracy.

1. Introduction

The analysis of inhomogeneous RS fields is a significant problem of the mechanics of deformable solid body. This problem has been of interest among many scientists since the beginning of the last century. However, in spite of it, a number of fundamental problems is not solved yet including development of experimentally verified adequate theoretical models of typical nonuniformly prestressed structures, building mathematical apparatus allowing to identify internal inhomogeneous RS distribution reliably in bodies in the framework of different concepts of nondestructive testing.

1.1. RS classification

RS exist in a body in the absence of external physical or mechanical actions. From the viewpoint of scales, RS are usually divided into the three following types in terms of the characteristic length $\mathfrak{L}$ [1,2]. The first type of RS is macro-stress: $\mathfrak{L}$ equals a scale of a whole structure. In this case the classical models of continuum mechanics are commonly used; polycrystalline and multiphase characters are ignored; the finite-element technologies are often employed to calculate such stresses. RS of the second type are micro-stresses balanced within several grains ($\mathfrak{L} = 3\mathfrak{g}$–$10\mathfrak{g}$, where $\mathfrak{g}$ is the characteristic grain size), e.g. internal-phase thermal RS in metal matrix composites. Finally, RS of the third type are balanced within the atomic size, inside a single grain ($\mathfrak{L} < \mathfrak{g}$), e.g. RS appearing due to dislocations and point defects.

1.2. RS origins

A presence of RS in solid bodies is typical for all the real objects. As a rule, RS state arises either during manufacturing processes (e.g. welding, hardening, heat treatment, rolling-and-pressing, etc.) or as a result of some loads, under elastic or viscoelastic deformation. RS components in different structures can reach relatively large values, especially in neighborhoods of concentrators like cracks, cavities, welding joints, and inclusions; in these cases, such stresses are generally difficult to observe and may cause destruction under loads being considerably less than permissible ones. One of the most important tasks for production engineers is to bring the RS level down, however, revealing the actual RS level in a construction is not always possible. Consideration of RS would allow to simulate engineering or biological structures of different complexity properly and to describe their behavior adequately in conditions of complex dynamical thermal-force load.

As for biological applications, it is worth noting that almost all the structures in human body are subjected to RS state – from living cells at micro-level to skin, bone tissues, and muscular tissues

* Corresponding author.
E-mail addresses: rdn90@bk.ru (R. Nedin), dudarev_vv@mail.ru (V. Dudarev), vatulyan@math.rsu.ru (A. Vatulyan).

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at macro-level. A lot of literature focus on exploration of RS state in solid biological structures of living organism, particularly in bone tissue. The extensive investigations on mechanical properties of bones taking into account RS state are carried out in the research works of I.F. Obraztsov, S. Yamada, A. Ahmed, B. McCormack and others. For instance, it was explored that RS in human cortical bones can reach up to 22 MPa [3]. When dealing with soft tissues with inherent large deformations, in order to simulate them, hyperelastic materials are often used in the framework of nonlinear elasticity theory. In the papers [4,5] some interesting results of such research are given. RS in vessels effect on a distribution of stresses and strains in deformed arterial walls in physiological state, and also on a thickness of arterial walls. Besides, it should be noted that when modeling such RS, a hypothesis of its homogeneity is often take on (e.g., see [6]). Many papers deal with studies of mechanical properties of various types of blood vessels; some theoretical, experimental and clinical principles are presented in research works of S. Cowin, J. Humphrey, G. Holzapfel, R. Ogden and others [7,4]. Thus, taking into account functional significance of RS from the viewpoint of mechanical effect on biological structures, when simulating them, it is necessary to take into consideration RS contribution to the total stress state of vascular system or bone tissue.

In addition, modeling of RS state in the mining mechanical engineering and geophysics are of certain interest. Consideration of inhomogeneous RS state in these problems would allow to correct a number of existing models, to simulate underground structures and pipelines more adequately, and to improve techniques of their diagnostics based on the acoustic methods [8].

1.3. RS modeling and identification

Selection of appropriate theoretical RS model is of great importance when formulating and solving specific practical problems. The evolution of mechanical models describing behavior of a body subjected to RS state started in the beginning of the XX century.

Although there are many different approaches to model RS state by means of the continuum mechanics apparatus, at present there is a lack of comprehensive review articles or monographs (one of the few articles is [9]) containing some RS models and common forms of corresponding constitutive relations with their comparison analysis and revealing the most efficient ones for some specific materials. At the same time, understanding which models adequately describe the presence of RS in bodies is vital.

In different literature published, when modeling and identifying RS state via nondestructive testing, they often use models with homogeneous RS. However, to provide efficient technologies of nondestructive testing, it is necessary to develop the models allowing to determine *inhomogeneous* RS state on the basis of known displacement or deformation fields at the body boundary with the usage of present-day computational algorithms including finite-element method [10–12]. One of the most efficient ways to get an additional data on concentrators and stress field in their regions is the acoustic method [13–16]. In the papers [17–26] some inverse problems on identification of various types of inhomogeneous RS states in beams, plates, cylinders and tubes are investigated on the theoretical basis of nondestructive acoustic method. In the monograph [8] some theoretical aspects of modelling and nondestructive identification of inhomogeneous RS in elastic bodies are presented. In the present paper, a comparative analysis of some RS models is carried out, and the efficient approaches to the reconstruction of inhomogeneous RS state in a thin plate are considered.

1.4. Brief historical review

Studies of problems with consideration of RS in mechanical and biological structures always rely on some theoretical RS models. The development of three-dimensional linearized theory of deformable bodies with RS began in the early XX century. Southwell was the first who derived the equations of linearized theory of stability in case of homogeneous subcritical state with small subcritical deformations (in 1913) [27]. This theory was afterward developed by Biezeno and Hencky [28], Biot [29], Neiber [30], and Trefftz [31]. In the year 1952, A.E. Green, R.S. Rivlin and R.T. Shield obtained the basic relations of linearized theory in the most general form for elastic isotropic body with arbitrary form of strain-energy function [32]. Later on, the models obtained by them were refined and used by Novozhilov [33], Truesdell [34], Tiersten [35], Washizu [36], Zubov [37], Guz [38], Hoger [39], Robertson [40], and others. One of the most comprehensive review on RS models for elasticity problems was made by Bažant in his paper [9]; let us also mention the work [41] by Zh.-B. Kuang containing a general RS model for the electroelasticity problem.

2. RS models

There are two approaches to the description of motion or equilibrium of prestressed bodies. Within the first one, the natural undeformed state of a body is considered as a reference configuration, and the constitutive relation contains the initial deformation in the explicit form. Within the second approach, the initial deformed configuration is taken up as a reference, and the reasons of RS formation are not taken into account so that the initial deformation does not enter into the equations explicitly. In the following Sections below, we review RS models in the framework of both approaches.

2.1. RS model without explicit consideration of initial deformation

Let us view an elastic body in two configurations (see Fig. 1). In the initial deformed configuration κ_0 the body is subjected to RS as a result of some finite deformation which is not taken into account here. We consider this configuration as reference one and use the Lagrangian material description of motion in terms of coordinates of the vector $\underline{R}_0 = (X_1^0, X_2^0, X_3^0)$. Let the body in this configuration have the volume V_0 bounded by the surface $S^0 = S_u^0 \cup S_\sigma^0$, where the initial displacements $\underline{U}^0$ are given at the boundary part S_u^0, and S_σ^0 is subjected to the initial traction $\underline{P}^0$. By perturbing the configuration κ_0 and superimposing small deformations on it, we get the second configuration κ, where the body is assumed to be of volume V, bounded by the surface $S = S_u \cup S_\sigma$, clamped at the boundary part S_u, and subjected to the periodic load $\dot{\underline{P}}$ at the boundary part S_σ. The linearized boundary problem has the following form:

$$\nabla_0 \cdot \underline{\underline{T}} + \rho_0 \dot{\underline{b}} = \rho_0 \frac{\partial^2 \underline{u}}{\partial t^2}, \tag{1}$$

$$\underline{\underline{T}} = \underline{\underline{\Gamma}}(\underline{\underline{\sigma}}_0, \underline{\underline{e}}, \underline{\underline{\omega}}) + \underline{\underline{L}}(\underline{\underline{e}}), \tag{2}$$

$$\nabla_0 \cdot \underline{\underline{\sigma}}_0 + \rho_0 \underline{b}_0 = 0, \tag{3}$$

$$\underline{u}_0|_{S_u^0} = \underline{U}_0, \quad \underline{n} \cdot \underline{\underline{\sigma}}_0\Big|_{S_\sigma^0} = \underline{P}_0, \tag{4}$$

$$\underline{u}|_{S_u} = 0, \quad \underline{n} \cdot \underline{\underline{T}}\Big|_{S_\sigma} = \dot{\underline{P}}, \tag{5}$$