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Plastic collapse of hardening spatial aluminium frames: A novel shakedown-based approach

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ABSTRACT

In the present work a novel approach is proposed for determining the safety factor of spatial aluminium frame structures against elastoplastic collapse. For the purposes of the study, a two-surface shakedown-based technique is developed, exploiting material's hardening features, tailored to the stress resultant plasticity discipline using linearized forms of codified criteria. This way, the classical advantages of direct methods of plasticity, such as robustness, mono-parametric safety assessment and load history independence are also enabled in the field of aluminium plastic design. Results demonstrate the effectiveness of the applied technique, providing a reliable alternative path for analysing inelastic performance of aluminium structures.

1. Introduction

Aluminium and its alloys represent a wide family of metallic materials, whose architectural appeal and special physical characteristics granted them increasing popularity and appreciation in construction activities throughout the years. Following chronologically their successful deployment in automotive industry and aircraft engineering, their adaptability in civil engineering works increased exponentially, despite their short history and recent introduction in this field, as compared to other traditional materials. The large number of alloys varying in manufacture process (wrought, cast) and temper (heat and non-heat treatable), as well as the form of available products (sheets/plates, bars, extruded sections), have attracted a growing interest from scientific community regarding material engineering and properties characterisation [1,2]. In addition, significant technological development in the last decades resulted in enlightening issues related to aluminium's structural feasibility, complementing lightness, corrosion resistance, and versatility of the cross sections configurations. This way, respective applications expanded, varying from non-structural systems (window frames, facades, curtain walls) to stress carrying members (beams, columns) and bearing structures (towers, footbridges) [3]. Intensive theoretical, numerical and experimental activity has been conducted in order to analyse their structural effectiveness, while issues related to local buckling and fatigue have been in the core of the research [4,5]. In last years, series of published material of educative character have addressed a wide variety of applications and guidelines at worldwide level [6–8]. Just recently, a trend for increasing familiarity with aluminium to national audiences among practice civil and architect engineers has been registered [9,10]. The epitome of this aluminium related upheaval is demonstrated through Eurocode 9 (EC9), the latest addition to the European codified provision series introduced by aluminium sector and standardization units, corresponding to the need to provide design specifications in construction [11].

However, the effectiveness and competitiveness of aluminium as constructional material depends significantly on the accurate prediction of its actual behaviour, which in the inelastic range is still characterized by complexity. Material's high non-linearity, the unavoidable mechanical and geometrical imperfections which are linked to the fabrication process, as well as strain hardening and the welding effects, make plastic design assessment mostly a cumbersome procedure. In addition, there are a lot of alloys with various characteristics that prevent them from exhibiting a homogeneous performance, thus influencing the structural behaviour in a unique way for each one. It is noted that even properties of the same alloy differ under various manufacture and heat treatment procedures [12].

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Nomenclature

HAZ	Heat Affected Zone
FEM	Finite Element Method
ESD	Elastic Shakedown
LA	Limit Analysis
PSD	Plastic Shakedown
ELM	Elastic Limit
N-M_y-M_z	interaction criterion for sections subjected to axial load and biaxial bending moment
Ω	3D spatial aluminium frame
n_E	number of column-beam finite elements
n_G	number of Gauss points
$\boldsymbol{L}^F$	convex polytope of external loads
n_F	number of vertices of $\boldsymbol{L}^F$
$\boldsymbol{v_o}$	constant loading vector
$\boldsymbol{v}^i$	variable loading vector
$\boldsymbol{s_o}$	stress-resultant vector, elastic structural response to $\boldsymbol{v_o}$
$\boldsymbol{s}^i$	stress-resultant vector, elastic structural response to $\boldsymbol{v}^i$
α	load scaling factor
$\boldsymbol{\sigma}$	elastic stress-resultant vector
$\boldsymbol{u}$	total elastoplastic stress-resultant vector
$\boldsymbol{\rho}$	residual stress field
$\boldsymbol{\pi}$	back-stresses vector
$\mathbf{H}$	equilibrium matrix of the structure
$\boldsymbol{F}_Y$	local yield criterion / yield surface
$\boldsymbol{F}_U$	local ultimate failure criterion / ultimate surface
$\boldsymbol{F}_{aux}$	auxiliary surface restricting the development of back-stresses
N	axial force
V_y	shear force, y axis
V_z	shear force, z axis
EC9	Eurocode 9/EN1999-1-1
LKH	Limited Kinematic Hardening
LP	Linear Programming
MRF	Moment Resisting Frame
ULS	Ultimate Limit States
SLS	Serviceability Limit States
EC1	Eurocode 1/EN1991
M_x	torsional moment
M_y	bending moment, y-y axis
M_z	bending moment, z-z axis
n	ratio of axial force over axial section capacity
m_y	ratio of bending moment over bending moment section capacity (y-y axis)
m_z	ratio of bending moment over bending moment section capacity (z-z axis)
$\boldsymbol{r}$	reduced stresses vector
$\boldsymbol{t}$	translation vector
r_k	radial coordinate (polar coordinate system)
φ_k	angular coordinate (polar coordinate system)
k	angular discretization index
n_K	number of sampling angular divisions
N_{el}	cross-section axial capacity on the elastic limit state
M_{el}	cross-section bending capacity on the elastic limit state
N_{pl}	cross-section axial capacity on the plastic limit state
M_{pl}	cross-section bending capacity on the plastic limit state
f_o	characteristic value of 0.2% proof strength
$\rho_{o.haz}$	ratio between 0.2% proof strength in HAZ and in parent material
f_u	characteristic value of ultimate tensile strength
$\rho_{u.haz}$	ratio between ultimate strength in HAZ and in parent material

Plastic theory implementation for designing aluminium alloys and its suitability were primarily studied in the work prepared by Ghaswala [13]. Providing an extensive literature survey on the plastic design aspects reflecting the trends of that time, the study highlighted the advantageous role of the stress-strain curve peculiar shape in enabling aluminium members to carry relatively high loads referring to redundant systems. Aluminium's behaviour in the post-elastic range is significantly influenced by the strain hardening feature of the material and the actual available ductility, which can sometimes pose a limitation to the full development of the expected collapse mechanism [14]. Despite inherent similarities with steel, they exhibit a general relationship of round-house type, which cannot be interpreted through the classic elastic-perfectly plastic idealization. In particular, this idealised model becomes inaccurate due to non-linearity of the stress-strain response below the yield point and considerable strain hardening beyond the yield point [15]. Furthermore, weakening of the metal around welds in case of aluminium modifies significantly the material properties in this area. The presence of this heat affected zone (HAZ) leads to strength reduction in the inelastic range which need to be considered [16]. Even with smaller welds, as used in thin members, the extent and severity of the HAZ effect is relatively significant.

For the inelastic analysis, a new approximated method has been worked out for practical purposes, being based on the generalization of the plastic hinge method. Based on the work prepared by Mandara & Mazzolani [17], a suitable adjustment of the plastic hinge method was proposed, in order to permit a reliable application of such method to the structures made of hardening materials and to this purpose, a correction factor η for the yield stress was introduced. With respect to codified provisions, Eurocode 9 provides methods and procedures referring to the plastic design of aluminium alloy structures through classifications of cross-sections based on the results of experimental data, as well as methods for the calculation of internal actions and the evaluation of member rotational capacity [18]. As stated in EC9-clause 6.1.4, plastic global analysis may be used only when member cross sections satisfy requirements specified for Class 1 cross sections and provided that the aluminium alloy exhibits sufficient ductility. The design rules for the evaluation of internal actions have been given by considering the actual behaviour of the material by means of different degree of refinement in the model of stress-strain relationship, related also to the type of alloys. The analysis of the global performance can be done at different levels from the simplest (linear elastic) to the most sophisticated (generally inelastic with strain hardening) giving rise to different degrees of reliability. In view of material law characterisation, codified provisions dictate that actual strain hardening behaviour of the alloy is being considered. To this purpose a variety of analytical models are provided in Annex E of EC9, ranging from simplified models, namely piecewise, bi or three-linear, with and without hardening, to more sophisticated ones in the form of continuous models, according to the Ramberg-Osgood law [19].

Essential progress has been conducted recently regarding the plastic design of aluminium alloy structures and modifications in codified provisions are proposed. In particular, in the scientific work by De Matteis et al. [20] the numerical investigation of aluminium extruded beams of I-cross sections of different treatment alloys were numerically investigated highlighting the beneficial effect on the inelastic response in case of non-heat-treated alloys and the recommendation for less restrictive slenderness limits than the one corresponding to heat-treated alloys, characterised by a weaker hardening. Moreover, through a numerical study, an alternative approach for the partial revision of numerical factor