Centres of rigidity in multi-storey asymmetric diaphragm systems for general lateral static loading

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A B S T R A C T

In this paper a general methodology for calculating the centres of rigidity in multi-storey asymmetric buildings is proposed, which is applicable to orthogonal as well as to non-orthogonal diaphragm systems with complex topology and possible coupling of diaphragms along with consideration of very general load conditions, including gravity and modal loads. The methodology is presented in two alternative forms: (a) In the standard form, matrix relations have been developed which can be directly included to any new or existing programming codes for analysis of building structures, (b) In the alternative form the methodology can use the results of any standard programs for static analysis of building structures, in addition with a small post-processing effort that can be achieved by only few hand calculations, thus becoming significantly simpler than the methodologies that currently exist. A numerical example presents the individual steps of a comparative application of the standard and alternative forms of the methodology, while two additional numerical examples demonstrate the simplicity and efficiency of its alternative form.

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1. Introduction

During ground seismic motions, multi-storey buildings with structural irregularities in their floor plans or along their height, generally vibrate in a lateral-torsional manner with significant swaying and twisting. Under this coupled lateral-torsional motion, structural members located along the perimeter of the buildings develop increased deformations and stresses due to buildings’ twisting, thus resulting in a higher risk of collapse.

For these reasons, several analytical efforts have been made by researchers to study this complex deformation and to analyze it in a superposition of simpler ones, composed of purely translational and purely torsional deformation patterns. In many of these attempts, various forms of the equivalent static analysis procedure have been used, because of their simplicity and acceptance by the seismic design codes. The main reason for using such static analyses today, is not so much the easy static solution of a simplified structural model, but the ambition for reliable substitution of dynamic seismic response analysis results by proper combinations of static analysis results. A major question in such an attempt is the adequate estimation and representation of the lateral–torsional coupling, by applying appropriate storey-torsional moments, usually obtained by proper static eccentricities from a suitable reference point.

Direct and accurate answers to this question can be achieved when finding and using the Centres of Rigidity (CR) of a multi-storey building [1–6], in order to calculate the proper static eccentricity of each floor as the distance between the centre of mass and rigidity of that floor. In the majority of the presented methodologies on this subject, several restrictive assumptions on the geometry of the building were made (“orthogonal” lateral resisting system, “proportional framing”, etc.).

Indirect approximate answers to this question, which bypass the calculation of rigidity centres, have been alternatively given, using the concept of finding an axis for minimal torsional effects (“optimum torsion axis”) as an origin for calculating static eccentricities [7–11]. It is noted that the methodologies using the concept of “optimum torsion axis” are applicable to buildings without any restrictions on stiffness distributions (such as “orthogonal” lateral resisting system, etc.). An extended bibliography on earthquake induced on torsion in buildings can be found in [12].

1.1. Definitions for centres of rigidity

The concept of centre of rigidity (CR), or centre of stiffness, arose from considering the behaviour of single-storey buildings...
with a rigid roof diaphragm [13]. It is defined as the point through which the resultant of the lateral load acting on a diaphragm passes, without causing any rotation at the diaphragm.

The extension of this concept to multi-storey asymmetric buildings is not trivial, since there are several definitions for the centres of rigidity [3]. Reviewing the literature, there is no generally accepted definition of the centres of rigidity for multi-storey buildings. Humar [14] interpreted the “single floor” CR as the point through which the resultant lateral forces at that floor can pass without causing rotation at that floor. The other floors may or may not have rotations. The calculation of CR according to this definition has been adopted in certain building analysis software (ETABS, etc.).

Cheung and Tso [3] suggested that to obtain the floor eccentricities, generalized centres of rigidity (“all floor CR”) can be defined as the set of points located at the floor levels such that when the given equivalent static lateral loads are applied through these points, no rotations at any of the floors will occur. In a subsequent paper by Tso [15], the difference between floor and storey eccentricities is explained as well as their dependence on the vertical distribution of the lateral seismic forces. In the present paper only the definition of “all floor CR” will be used. The calculation of CR according to this definition has not been yet adopted in building analysis software.

1.2. Existing methodologies for calculating centres of rigidity

Reviewing the existing methodologies for the calculation of “all floor CR”, Cheung and Tso [3] made the assumption that the multi-storey building has an orthogonal arrangement of lateral-load resisting elements (orthogonal framing), connected by rigid floor diaphragms. The principal axes of the vertical elements are parallel to the orthogonal framing, while 2 sets of lateral loads $P_x$ and $P_y$ have been considered, that are respectively parallel to the horizontal axes of the orthogonal framing system. Using a matrix procedure based on global stiffness sub-matrices of the building, they showed that CRs are load dependent, namely the x-coordinates of CRs are functions of $P_y$, while the y-coordinates of CRs are functions of $P_x$. They also showed that in the case of orthogonal and also proportional framing system, the locations of CRs are load independent and lie on the same vertical line. The methodology requires the formulation of stiffness sub-matrices and load vectors, together with matrix inversions and other operations. Also an alternative procedure was suggested, which requires the use of any standard plane-frame software for 2 proper static analyses of the building, together with the post-processing of the results.

Hejaj and Chopra [4], proposed a matrix procedure for calculating the centres of rigidity of multi-storey asymmetric buildings with orthogonal or non-orthogonal framing. Using 2 sets of lateral loads $P_x$ and $P_y$ that are parallel to the horizontal axes of the global coordinate system, they also affirmed the conclusion of Cheung and Tso [3], that the x-coordinates of CRs are functions of $P_y$, while the y-coordinates of CRs are functions of $P_x$. Their formulation, based on global stiffness sub-matrices of the building, cannot be easily applicable when using standard building analysis software, since formulation of stiffness sub-matrices and load vectors is required, together with matrix inversions and other operations and the post-processing of the results.

Goel and Chopra [16] showed that code provisions for eccentric application of lateral static loads to multi-storey buildings can be carried out using a method which combines results of 3 additional static analyses (obtained by any commercial building analysis software), without having to compute rigidity centres in the various floors. The methodology can be applied to buildings with orthogonal arrangement of lateral-load resisting elements connected by rigid floor diaphragms, for 2 sets of lateral loads $P_x$ and $P_y$ that are parallel to the horizontal axes of the global coordinate system. In addition, the simplified assumption was made that for lateral force analysis along y (x) direction, the building’s plan is treated as symmetric about the x (y) axis. The results post-processing of 3 different static analyses make necessary the additional use of a specific computational software or a spreadsheet.

Basu and Jain [5] proposed a matrix procedure for calculating the centres of rigidity of asymmetric buildings with non-orthogonal framing, using 2 sets of lateral loads $P_x$ and $P_y$ that are respectively parallel to the horizontal axes of the global coordinate system. Their methodology requires the formulation of a flexibility matrix and 2 displacement vectors of the global structure, $(3N + 2)$ static solutions to form the flexibility matrix and the displacement vectors of the structure, and the use of application formulas to find the rigidity centres. They also affirmed the conclusions of Cheung and Tso [3] and Hejaj-Chopra [4], that the x-coordinates of CRs are functions of $P_y$ and the y-coordinates of CRs are functions of $P_x$. However their methodology presents unnecessary complexity, since the calculation of rigidity centres is achieved with respect to mass centres of the floors, while such a dependency does not physically exist, leading to unreliable results in the presented example (the rigidity centre of the top floor resulted to be in a different position than the shear centre of the top storey). Furthermore the methodology requires additional results post-processing of standard building analysis software, that cannot be performed by hand calculations.

Bosco et al. [6] proposed a methodology for evaluating the in-plan irregularity of non-regularly asymmetric buildings, which includes calculation of the position of floor stiffness centres (CRs) and of the ratio of uncoupled torsional to lateral frequencies. The presented methodology can be applied when the principal axes of the building are a priori known (mono-eccentric buildings, orthogonal framing, etc.), and requires a little post-processing effort on the analysis results due to four sets of horizontal forces and torsional couples. The methodology provides rigorous results for the position of CRs in the case of regularly asymmetric systems but provides approximate results in the case of non-regularly asymmetric ones.

At this point it is noted that several of the existing papers ([2,4,15,17], etc.) for calculating rigidity centres of multi-storey asymmetric buildings, as well as several Seismic Codes (Eurocode 8, etc.), maintain that these centres cannot be calculated for every building, thus resulting in possible confusion for those who do not know well this specific subject. The right wording is that the load-independent rigidity centres of multi-storey asymmetric building exist only for some special classes of buildings (with “proportional framing”, with 2 horizontal axes of symmetry, etc.), which means that for all the remaining building classes the rigidity centres are load dependent in general.

The above presented existing methodologies for calculating the CR of multi-storey asymmetric buildings, present several deficiencies when a practicing engineer wants to use any of them:

- Matrix formulations, inversions and other matrix operations must be performed when using global stiffness sub-matrices [3,4].
- Matrix formulations, other matrix operations and $(3N + 2)$ static solutions are required when flexibility matrix procedures are used [5].
- The 2 sets of lateral loads that are used, are respectively parallel to the horizontal axes Ox and Oy of the global coordinate system Ox翟. Other kinds or directions of loads have not been considered. However, in multi-storey asymmetric buildings, inertia loads (like the modal loads of any specific eigen-mode) are not necessarily parallel to each other.