Engineering Structures 150 (2017) 166–175

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

# Inverse problems in structural safety analysis with combined probabilistic and non-probabilistic uncertainty models

K. Karuna, C.S. Manohar *

*Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India*

ARTICLE INFO

*Article history:*
Received 29 November 2016
Revised 1 June 2017
Accepted 14 July 2017

*Keywords:*
Structural safety
Inverse reliability
Convex functions
Reliability based structural optimization

ABSTRACT

In problems of structural safety analysis, depending upon the nature and extent of availability of empirical data, uncertainties could be quantified by using probabilistic and/or non-probabilistic models. The options for non-probabilistic representations include intervals, convex functions, and (or) fuzzy variable models. We consider a few inverse problems of structural safety analysis aimed at the determination of system parameters to ensure a target level of safety and/or to minimize a cost function for problems involving combined probabilistic and non-probabilistic uncertainty modeling. The treatment of this problem calls for combining methods of uncertainty analysis with finite element structural modeling and numerical optimization tools. Development of load and resistance factor design format, in problems with combined uncertainty models, is also presented. We employ super-ellipsoid based convex function models for representing non-probabilistic uncertainties. The target safety levels are taken to be specified in terms of indices defined in standard space of uncertain variables involving standard normal random variables and/or unit hyper-spheres. A class of problems amenable for exact solutions is identified and a general procedure for dealing with more general problems involving nonlinear performance functions is developed. Illustrations include studies on inelastic frame with uncertain properties. Accompanying supplementary material contains additional illustrations.



## 1. Introduction

The forward problem of structural safety assessment consists of quantifying a measure of safety given a performance function and a mathematical model for the uncertainties relevant to the problem. Extensive studies on this class of problems have been reported in the existing literature for the case when uncertainties are modelled using probabilistic framework [1–3]. These models work well when the empirical data needed to model the uncertainties are available adequately, and satisfy the requirements of being random samples. When these conditions are not met, alternative modeling frameworks (for example, those based on theories of intervals, convex functions, and fuzzy variables) become appropriate and these have been explored in the engineering mechanics literature [4–14]. The monograph by Moller and Beer [15] extensively covers methods for uncertainty modeling using fuzzy random variables in the context of civil structures and computational mechanics. In the broader context of science and engineering, the fact that, both aleatoric and epistemic uncertainties typically co-exist while modeling a given physical variable, has been recognized, and, this has lead to several modeling frameworks, such as, probability box, random sets, evidence theory, fuzzy random variables, and the more general polymorphic models for uncertainty [see, for example, 16–24]. The importance of these developments has been recognized for problems of engineering mechanics and this is evidenced by thematic issues of research journals focusing on these topics [25–28].

The inverse safety analysis problems, on the other hand, consists of determining one or more of the system parameters (which could be deterministic system parameters and (or) parameters of an uncertainty model) when a target level of safety is specified, or, more generally, to minimize a cost function with constraints on safety measures. Such studies, for the case when the uncertainties are characterized probabilistically, include

* Corresponding author.
*E-mail addresses:* karuna@civil.iisc.ernet.in (K. Karuna), manohar@civil.iisc.ernet.in (C.S. Manohar).

http://dx.doi.org/10.1016/j.engstruct.2017.07.044
0141-0296/

the development of load and resistance factors which aid reliability-based design [1,3,29–31], procedures based on first order reliability method (FORM) [32–34], and reliability-based design optimization method which minimize specific cost functions under the constraints of target reliability with respect to specified limit surfaces [35]. The papers [36–37] present extensive reviews of literature pertaining to problems of reliability based structural optimization.

In the context of non-probabilistic, or, combined probabilistic and non-probabilistic description of uncertainties, studies in the existing literature addressing problems of inverse safety analysis are very limited. The study [8] has shown the common threads that exist in the problems of safety assessment when uncertainties are specified using alternative frameworks (viz., random variables, intervals, convex functions and fuzzy variables). In all these contexts, the study has shown that a measure of safety can be obtained by solving a constrained nonlinear optimization problem as in structural reliability studies. The recently published series of papers [38–43] considers options for non-probabilistic modeling of limited empirical data based on theories of intervals and convex functions (ellipsoids and super ellipsoids). These studies contain illustrations on skeletal structures as well as finite element based structural models. Questions on how to decide on choice of non-probabilistic model, and, how newly acquired data can be incorporated into an existing convex model are also addressed. Problems of design optimization, in which, uncertainties are modelled by using combined probabilistic and convex function models, have been discussed in [44]. Computational issues arising in problems of structural optimization involving non-probabilistic and (or) probabilistic uncertainty models have been explained using perturbational strategies [45], surrogate modeling methods [46], and sequential optimization strategies [47]. The study [48] has considered problems of reliability based design optimization in which the uncertain variables are characterized in terms of random variables with interval valued distributional parameters.

The present authors have recently discussed the use of ellipsoids/super-ellipsoids with minimum volumes, and, convex functions modelled as contours of Nataf's probability density function (pdf), in modeling uncertainties using convex function and fuzzy variable approach [14]. In this context, these authors have introduced the idea of a unit hyper-sphere as the standard uncertainty space and have shown that the problem of quantifying safety measure in a wide range of contexts (involving alternative uncertainty modeling frameworks) reduces to an identical form, namely, that of finding the shortest distance from the origin of an unit hyper-sphere to the limit surface in the standard transformed space. The present study aims to extend the idea of standard space introduced in this earlier work to address problems of inverse safety analysis when uncertainties are modelled, in general, using combined non-probabilistic and probabilistic frameworks. The study specifically considers the following three problems: (a) development of load and resistance factor design (LRFD) design format, (b) determination of a set of design variables so as to achieve a target level of safety, and (c) determination of a set of design variables which minimize a cost function under the constraints on safety measures. Illustrations include studies on linear/nonlinear beam and simple frame structures. It is pointed out that, while the study allows for both probabilistic and non-probabilistic modeling frameworks to be employed within the ambit of a single problem, the question of treating simultaneous presence of aleatoric and epistemic uncertainties with respect to a single uncertain variable (via the use of polymorphic models of uncertainties), on the other hand, has not been addressed.

## 2. Problem statement

Consider the problem of structural safety analysis in which the uncertainties in specifying load and structural parameters are collectively represented through a $n \times 1$ vector $\boldsymbol{Q}$. We consider the situation in which the empirical data available to model $\boldsymbol{Q}$ could be inadequate, to varying degrees, and a probabilistic model encompassing all components of $\boldsymbol{Q}$ may not be possible. Consequently, a modeling framework involving a combination of random variables, intervals, convex functions and fuzzy variables is proposed to be employed. To allow for this, we write $\boldsymbol{Q} = (\boldsymbol{\Theta} \quad \boldsymbol{X} \quad \boldsymbol{\Psi})^t$ such that $\boldsymbol{\Theta} = n_{\boldsymbol{\Theta}} \times 1$ vector of random variables, $\boldsymbol{X} = n_{\boldsymbol{X}} \times 1$ vector of variables modelled using a convex function $\Omega(\boldsymbol{X})$ such that $\boldsymbol{X} \in \Omega(\boldsymbol{X})$, and $\boldsymbol{\Psi} = n_{\boldsymbol{\Psi}} \times 1$ vector of fuzzy variables characterized in terms of the membership function $\Omega_f(\boldsymbol{\Psi}, \alpha); 0 \leqslant \alpha \leqslant 1$; it is noted that $n_{\boldsymbol{\Theta}} + n_{\boldsymbol{X}} + n_{\boldsymbol{\Psi}} = n$. Let $g(\boldsymbol{Q})$ denote a scalar valued performance function such that the region in the space spanned by $Q_1, Q_2, \cdots, Q_n$ in which $g(\boldsymbol{Q}) < 0$ and $g(\boldsymbol{Q}) > 0$, respectively, denote the unsafe and safe regions with the surface $g(\boldsymbol{Q}) = 0$ representing the limit surface. The specification of elements of $\boldsymbol{Q} = (\boldsymbol{\Theta} \quad \boldsymbol{X} \quad \boldsymbol{\Psi})^t$ is taken to be as follows:

(a) The vector $\boldsymbol{\Theta}$ is specified through its $n_{\boldsymbol{\Theta}}$-the order joint pdf $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$. In case the complete specification of $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$ is unavailable, it is assumed that the first order pdf-s, $p_{\Theta_i}(\theta_i); i = 1, 2, \ldots, n_{\boldsymbol{\Theta}}$ and the $n_{\boldsymbol{\Theta}} \times n_{\boldsymbol{\Theta}}$ matrix of correlation coefficients are available so that a Nataf's model for the joint pdf can be constructed. Using standard methods of transformations, the vector $\boldsymbol{\Theta}$ can be transformed to the vector of standard normal random variables denoted by $\boldsymbol{U}$. One could, in principle, consider more general models for representing probability distribution of $\boldsymbol{\Theta}$, such as, those based on P-boxes, or, the more detailed polymorphic uncertainty models. As has been already noted, the present study does not consider these options.

(b) The specification of the convex model for $\boldsymbol{X}$ follows the procedure described by [14]. These authors have considered two alternatives for representing $\Omega(\boldsymbol{X})$: the one based on super-ellipsoids and the other based on convex regions fashioned after contours of a Nataf's pdf. In the present study, we limit our attention to the first of these alternatives. Box 1 summarizes the relevant steps involved. As may be observed, the vector $\boldsymbol{X}$ here is transformed to $\boldsymbol{Z}$ so that, in the transformed space, the convex region $\Omega(\boldsymbol{X})$ gets mapped to a unit hyper-sphere. It is also noted that with $n_i \approx 4$, $i = 1, 2, \cdots, n_{\boldsymbol{X}}$, the convex region reasonably well approximates an $n$-dimensional hypercube, thus providing a means to deal with interval models. In view of this, in the present study, the interval modeling strategies are not separately considered.

(c) The specification of fuzzy variables $\psi_i, i = 1, 2, \cdots, n_{\boldsymbol{\Psi}}$ is through a membership function $0 \leqslant \mu(\psi_1, \psi_2, \ldots, \psi_{n_\Psi}) \leqslant 1$ such that for any $\alpha \in [0, 1]$, the function $\mu(\psi_1, \psi_2, \ldots, \psi_{n_\Psi}) = \alpha$ is a convex function in the space of $\psi_i, i = 1, 2, \cdots, n_{\boldsymbol{\Psi}}$. Thus, the fuzzy variable model can be interpreted as a parametered set of convex functions. Consequently, the procedure for transforming the fuzzy variable to standard space of nested hyper-spheres, with radii varying between 0 and 1, follows the procedure as has been used in the case of convex functions. Here the convex function corresponding to the $\alpha$-cut with $\alpha = 0$ is mapped to a hyper-sphere of unit radius and the remaining functions corresponding to $\alpha$-cut with $0 < \alpha \leqslant 1$ are mapped to inner hyper-spheres with radii lesser than unity.