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A new model for beam crack detection and localization using a discrete model

Ahmed Khnaijar *, Rhali Benamar

Laboratoire des Etudes et Recherches en Simulation, Instrumentation et Mesures (LERSIM), Mohammed V University of Rabat-Mohammadia School of Engineers, Avenue Ibn Sina, BP 765, Rabat, Morocco

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ABSTRACT

This paper presents a new discrete physical model to approach the problem of cracked beam vibrations. The model consists of a beam made of several small and evenly spaced bars. The beam bending stiffness is modeled by spiral springs that serve also as a crack model. Concentrated masses presenting the inertia of the beam are located at the bar ends. This model has the advantage of simplifying parametric studies, because of its discrete nature, allowing easy modification in the crack position and magnitude. Therefore, once the model is established, various practical applications may be performed without the need to go through all the formulations again. As a result, this model allows conducting a parametric study with the objective of facilitating the diagnostics process involving both crack localization and depth estimation.

1. Introduction

The vibration analysis of cracked beams is of a great interest in many fields, especially in civil and mechanical engineering. Various methods applied to the modeling of such problems have been addressed in the literature. A discrete model, such as the one presented in this paper, may be useful in multiple engineering applications due to its adaptability and ability to simplify the parameters variation.

Discretization method, such as the Finite-Element Method (FEM), is one of the most commonly used methods for analysis of large structures [1–4]. However, continuous models may be more efficient for a single beam. In general, the crack model is presented as a loss in stiffness in the crack location, where the stiffness variation is essentially due to stress concentration in the crack vicinity [5,6]. Dimagoronas, in his state of the art [7], detailed the evolution of cracked beam models. In the mid-twentieth century, Irwin [5], Bueckner [8], Westmann and Yang [9] linked local flexibility to the crack stress intensity factor (SIF). Thus, by analyzing experimental results, they established a method for calculating the SIF based on the local bending stiffness (the inverse of the local flexibility) of a cracked rectangular beam.

Crack modeling is important in establishing a relationship between the crack location and depth, and the structural response. Chondros and Dimarogonas [10,11], and Dimarogonas and Massouros [12] developed a frequency spectral method for the identification of a crack based upon the results of fracture mechanics and the spiral spring model. This method established a link between the crack depth and the change in the natural frequencies of the first three harmonics of the structure for a known crack position. Moreover, an experimental technique for crack identification using natural frequencies was developed by Adam and Cawley [13,14]. Yang [15] proposed a method for crack detection based on the superposition of the frequencies obtained from the crack frequency response for two to three modes. More recently, Barad et al. [16], Gillich and Praisach [17], and Maghsoodi et al. [18] improved the crack detection procedure based on the frequency response function. On the other hand, Nguyen [19] introduced a new method for crack detection using the distortion occurring in the mode shape around the crack position.

In this paper, a theoretical discrete model is used to approach cracked beam vibration. The beam is presented by bars, masses, and spiral springs. Since the model is composed of a multitude of springs, each one may correspond to a crack. The cracks locations, number, and magnitude may be modified at will. Section 2 introduces the general theory, including details of the discrete model

* Corresponding author at: Villa 5, Lot. AL Mountalak 1, Hay Al Fath, C.Y.M, Rabat, Morocco.

E-mail addresses: khnaijar@gmail.com (A. Khnaijar), rhali.benamar@gmail.com (R. Benamar).

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Nomenclature

a	crack depth
a_j	contribution coefficient of the j^{th} linear mode shape in DB for the discrete system
$\{\mathbf{A}\}$	displacement amplitudes of the masses $m_1; \ldots; m_i; \ldots; m_N$ in DB
C_r	the stiffness coefficient of the r^{th} spiral spring
C_i^c	the stiffness coefficient of the i^{th} crack
DB	displacement basis
δ_{ij}	the Kronecker's symbol
$\{\mathrm{d_i}\}$	the i^{th} displacement vector in DB
E	the modulus of elasticity or the Young modulus of the bar's material
h	the thickness of the bar in (m)
I	the second moment of area relative to the neutral fibre for the cross section in (m^4)
I_{cr}	the second moment of area of the reduced beam relative to the neutral fibre for the cross section in (m^4)
$[\mathbf{I}]$	the identity matrix
k_{ij}^s	the general term of the linear rigidity tensor in DB (spiral spring)
$[\mathbf{K}^s]$	matrix of linear rigidity in DB (spiral spring)
$[\mathbf{K}_N^{SS}]$	linear rigidity matrix of the N-DOF discrete system presenting a simply supported beam
$[\mathbf{K}_N^{CC}]$	linear rigidity matrix of the N-DOF discrete system presenting a clamped–clamped beam
$[\mathbf{K}_{N-2}^{CC}]$	linear rigidity matrix of the N-2-DOF discrete system presenting a clamped–clamped beam
$[\mathbf{K}_N^{CF}]$	linear rigidity matrix of the N-DOF discrete system presenting a clamped–free beam
$[\mathbf{K}_{N-1}^{CF}]$	linear rigidity matrix of the N-1-DOF discrete system presenting a cantilever beam
l_i	length of the ith bar in (m)
L	total length of the beam
m_{ij}	the general term of the mass tensor in DB
$[\mathbf{M}]$	matrix of masses in DB
$\mathcal{M}$	moment in the spiral spring
N	number of degrees of freedom (number of masses)
S	the area of cross section for uniform beam in (m^2)
T	the kinetic energy
V	the total potential energy
V_l^s	the beam bending strain energy
V_l	the linear potential energy
W	beam transverse displacement
c	the crack distance on the beam
y_i	transverse displacement of the ith mass in DB
β_i	the ith eigenvalue of the rigidity
θ_r	the angular displacement of the adjacent bars at the rth node in (rad)
ρ	mass per unit volume of the bar (kg/m^3)
ξ_i	the dimensionless crack depth ratio at the ith position.
ω_{discr}^{SS}	the frequency of the discrete system in the case of simply supported beam
ω_{discr}^{CC}	the frequency of the discrete system in the case of clamped beam
ω_{discr}^{CF}	frequency of the discrete system in the case of cantilever beam

assumptions, and parameters calculations. Section 3 is devoted to the validation of the crack model for simply supported, clamped, and cantilever beams. Finally, Section 4 presents a crack detection procedure based on the present model.

2. General theory

A discrete model of a beam consisting of bars, concentrated masses, and rotational springs located at the bar ends was presented by Rahmouni et al. [20] to approach the vibrations of a continuous beam using different assumptions. Since the models of fracture mechanics present cracks as spiral springs, this approach may be useful in establishing a new model to study the vibration of cracked beams.

2.1. General formulation

A crack is an opening that usually occurs when an element reaches its tensile limit. Therefore, it may or may not be accompanied by a loss of matter. Thus, to approach the crack macroscopically, Fig. 1, the beam is presented as a series of small bars where the rth bar, corresponding to the crack, has a smaller depth.

By adapting the model introduced in [20] for nonlinear vibrations of uncracked beams, a new model is developed here for beams with n cracks located at n different positions along the beam. The present model consists on the N-degree-of-freedom discrete model shown in Fig. 2, with N masses $m_1, \ldots, m_N$, located at the ends of $(N+1)$ rigid bars, connected by $(N+2-n)$ spiral springs simulating the beam bending stiffness. The n cracks are modeled by n spiral springs simulating the cracks reduced stiffness, estimated using the model presented in Fig. 1. The stiffness of the r^{th} spring is denoted by C_r, for $r = 1$ to $(N+2-n)$), and the stiffness coefficient of thei^{th} spiral spring, presenting the i^{th} crack, is denoted by C_i^c, for $i = 1$ to n. The bending moment $\mathcal{M}$ in the r^{th} spiral spring connecting the bars $(r-1)$ is given by: $\mathcal{M} = -C_r\Delta\theta$; $\Delta\theta = \theta_r - \theta_{r-1}$ being the angle between the bars adjacent to the node r.

The displacement vector presenting the vertical displacements of the N masses, Fig. 2, may be written as follows:

$$\{\mathbf{y}\} = y_1\{\mathbf{d}_1\} + y_2\{\mathbf{d}_2\} + \ldots + y_i\{\mathbf{d}_i\} + \ldots y_N\{\mathbf{d}_N\} \qquad (1)$$

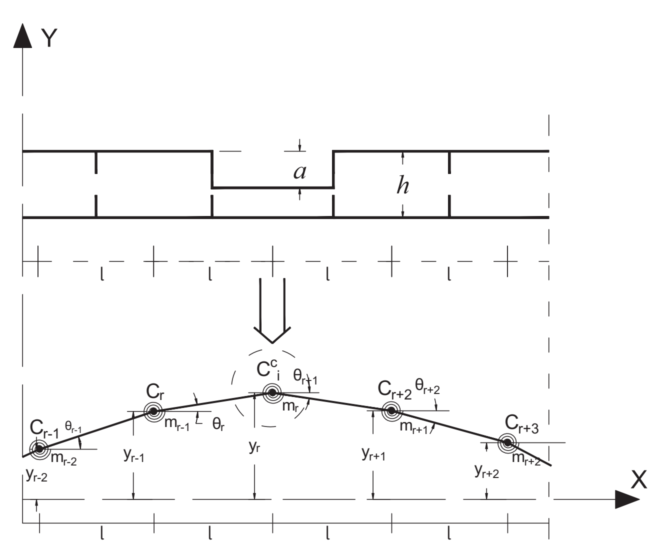

Fig. 1. crack model.