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A general spectral difference method for calculating the minimum safety distance to avoid the pounding of adjacent structures during earthquakes

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ABSTRACT

With the development of cities, buildings have become increasingly concentrated. Therefore, there are serious potential collision dangers between the adjacent buildings or the building partitions under earthquakes, hence buildings need to set the appropriate safety distance to avoid pounding. However, the calculation of the minimum safety distances (**MSD**) of the adjacent buildings in Chinese Code is rough and arbitrary, while the traditional response spectrum method is derived based on the assumption of proportional damping and it is difficult to consider the non-proportional damping characteristics for large and complex building systems. Based on the above, this paper proposes a new *general spectral difference method* (**gSDM**) to calculate the MSD. First, the pseudo-excitation method is used to derive the relative displacement random expression of adjacent buildings during earthquakes. Then, combined with the response spectrum method, the gSDM formula is established, in addition, the explicit expressions of combination coefficient in the gSDM formula are given. Finally, the examples of two adjacent multi-storey shear-buildings are used to illustrate the accuracy and wide adaptability of the gSDM during the El-Centro earthquake. The results compared and discussed in detail with the *classical spectral difference method* (**cSDM**), SRSS method and time history analysis.

1. Introduction

Since the 21st century, the process of city construction, especially in developing countries, has been growing rapidly. Much large building with various functions are constructed to meet the work and life requirements of the population and large and medium-sized buildings are becoming increasingly concentrated. Therefore, there are serious hidden collision dangers between the adjacent buildings or the building partitions under the earthquake. In recent years, many buildings damaged or collapsed during China's Wenchuan, Yushu and Yaan earthquakes from collisions. Previously, the research on seismic pounding has primarily focused on bridge structures [1,2], and building structures are usually simplified to a single degree of freedom model for building seismic pounding simulations [3,4]. While the response spectrum method with design guiding significance is established during this period [5]. Now, the research work has gradually focused on the following aspects, such as collision safety distance stochastic computing [6,7], establishing reasonable collision model [8] and control measures to prevent seismic pounding [9,10]. Considering the development of the times in civil engineering, some new type of vibration/isolation device, the new material and new structures are used, so a large number of building systems, such as buildings-damper system, the multi-material composite structure system, the building-bridge integrated system, and soil-structure interaction system, present more complex damping characteristics. The off-diagonal terms are ignored in traditional computing methods, i.e., the forced decoupling method (FDM). However, when a building has a remarkable non-proportional characteristic, the FDM method may lead to significant errors, which may cause unsafe seismic design of buildings. If a non-conservative computing result is obtained, the seismic design of the building is unsafe. Therefore, it is necessary to improve the traditional calculation methods and consider the non-proportional damping effect in the formulas to yield more accurate and reliable results.

At present, scholars have introduced non-proportional damping characteristics into the stochastic response spectrum calculation for single buildings and have proposed a variety of methods for

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solving the non-proportional damping matrix equations, such as the complex modal decoupling method [11], the matrix perturbation solution method [12] and iterative calculations method [13]. And further combined with the pseudo-excitation method [14], an improved stochastic calculation expression is proposed based on complex modal theory and iterative processes [15,16]. In order to improve the accuracy of response spectrum method in non-proportional damping structure, Antonio proposed an equivalent damping ratio calculation method to compute a building-damper system with remarkable non-proportional characteristics [17], which can more rational use of FDM. Zhou Xiyuan deduced a new response spectrum combination formula based on the random analysis of non-proportional structures that can be used in complex damping situations [18]. The above-mentioned two kinds of mainstream ideas can be directly taken from the traditional response spectrum curve, and obtain relatively accurate results. However, these research achievements are limited to individual buildings, and the seismic pounding of adjacent buildings is not considered. Guo Wei suggested an improved spectral difference method that accounts for collisions between stations and bridges in building-bridge integrated system [19], but the bridge still is simplified to a single DOF system. In summary, the stochastic response spectrum calculation of the seismic collision minimum safety distance between adjacent structures with significant non-proportional damping characteristics is still relatively deficient.

In view of this, a stochastic method to calculate the minimum safety distance to avoid pounding between adjacent structures is developed in this study. In addition, an improved accurate response spectrum method, namely *general spectral difference method* (gSDM), to directly calculate the safety distance between adjacent buildings for seismic design is proposed. And the explicit expressions of combination coefficient in the gSDM formula are given. Finally, through numerical examples demonstrate. In contrast to traditional methods, the gSDM is more accurate, more efficient and more convenient for practical seismic design in calculating the minimum safety distance between adjacent structures in earthquakes.

2. Seismic response of non-proportional damping structures

Based on structural dynamics theory, the numerical model of structures under unidirectional seismic excitation can be expressed as follows:

$$\boldsymbol{M}_s\ddot{\boldsymbol{U}}_s + \boldsymbol{C}_s\dot{\boldsymbol{U}}_s + \boldsymbol{K}_s\boldsymbol{U}_s = -\boldsymbol{M}_s\boldsymbol{E}_s\ddot{u}_g \tag{1}$$

where $\boldsymbol{M}_s$, $\boldsymbol{C}_s$ and $\boldsymbol{K}_s$ are the mass matrix, the damping matrix, and the stiffness matrix, respectively, which include the translational and torsional responses. The $\boldsymbol{U}_s = \left[\boldsymbol{U}_{sx}, \boldsymbol{U}_{sy}, \boldsymbol{U}_{s\theta}\right]^T = \left[u_{sx1}, \ldots, u_{sxn}, u_{sy1}, \ldots, u_{syn}, u_{s\theta 1}, \ldots, u_{s\theta n}\right]^T$ is the relative displacement vector of the structures from the ground. $\boldsymbol{E}_s$ is the seismic load position vector. $\ddot{u}_g$ is the absolute ground acceleration and is constant. For the actual building, the non-proportional damping is a universal phenomenon. Whereas the approximation proportional damping model, which the damping matrix $\boldsymbol{C}_s$ can be decoupled through the modal transformation of the undamped structures, is usually adapted in engineering. However, the actual damping characteristics of buildings are often more complex, such as the use of damping devices and isolation design for buildings vibration control, the assumption of proportional damping does not meet the actual requirements. So the damping matrix $\boldsymbol{C}_s$ in Eq. (1) cannot be decoupled with modal transformation. In this case, the state equation can be developed, and the complex mode theory can be utilized to achieve diagonalization decoupling operation.

First, define a state vector $\boldsymbol{U} = \left[\dot{\boldsymbol{U}}_s \quad \boldsymbol{U}_s\right]^T$ and introduce the identity formula $\boldsymbol{M}_s\dot{\boldsymbol{U}}_s - \boldsymbol{M}_s\dot{\boldsymbol{U}}_s = \boldsymbol{0}$ to build the state equation as follows:

$$\boldsymbol{A}\dot{\boldsymbol{U}} + \boldsymbol{B}\boldsymbol{U} = -\boldsymbol{A}\boldsymbol{E}\ddot{u}_g \tag{2}$$

where $\boldsymbol{A} = [\boldsymbol{0}, \boldsymbol{M}_s; \boldsymbol{M}_s, \boldsymbol{C}_s]$, $\boldsymbol{B} = [-\boldsymbol{M}_s, \boldsymbol{0}; \boldsymbol{0}, \boldsymbol{K}_s]$, and $\boldsymbol{E} = [\boldsymbol{E}_s; \boldsymbol{0}_{3n\times 1}]$. There are $3n$ groups of conjugate eigenvalues $\boldsymbol{\eta}$ and eigenvectors $\boldsymbol{\Phi}$ satisfying the following equation:

$$\boldsymbol{B}\boldsymbol{\Phi}_i = -\eta_i\boldsymbol{A}\boldsymbol{\Phi}_i \tag{3}$$

where $\boldsymbol{\Phi}_i = [\eta_i\boldsymbol{\varphi}_i; \boldsymbol{\varphi}_i]$, $\boldsymbol{\varphi}_i = \boldsymbol{\varphi}_{iR} + r\boldsymbol{\varphi}_{iI}$, $\eta_i = -\eta_{iR} + r_{iI} = -\xi_i\omega_i + r\omega_i\sqrt{1-\xi_i^2}$, $r = \sqrt{-1}$, and $i = 1 \sim 3n$, Different eigenvectors have generalized orthogonal relations among one another, such that the undamped frequency ω_i and the corresponding damping ratio ξ_i can be obtained through the orthogonal relation of the complex modal vector, as follows:

$$\omega_i^2 = \frac{(\boldsymbol{\varphi}_i^*)^T\boldsymbol{K}_s\boldsymbol{\varphi}_i}{(\boldsymbol{\varphi}_i^*)^T\boldsymbol{M}_s\boldsymbol{\varphi}_i}, \quad \xi_i = \frac{1}{2\omega_i}\frac{(\boldsymbol{\varphi}_i^*)^T\boldsymbol{C}_s\boldsymbol{\varphi}_i}{(\boldsymbol{\varphi}_i^*)^T\boldsymbol{M}_s\boldsymbol{\varphi}_i} \tag{4}$$

where the superscript "$*$" indicates the conjugate operation. Because the resulting $3n \times 2 = 6n$ eigenvectors are completely orthonormal ($3n$ groups of mutually conjugate vectors), the dynamic response of a non-proportional damping structure can be expressed in the form of a vector superposition as follows: $\boldsymbol{U} = \sum_{i=1}^{6n}\boldsymbol{\Phi}_iQ_i$, where Q_i is the i_{th} generalized displacement response of the structure system, reflecting the structural response components in the i_{th} mode. Substituting it into Eq. (2) and the front is multiplied by $\boldsymbol{\Phi}_i^T$, then results in Eq. (5) as follows:

$$\dot{Q}_i - \eta_iQ_i = -\mu_i\ddot{u}_g \tag{5}$$

where $\eta_i = -(\boldsymbol{\Phi}_i^T\boldsymbol{B}\boldsymbol{\Phi}_i)/(\boldsymbol{\Phi}_i^T\boldsymbol{A}\boldsymbol{\Phi}_i)$ and $\mu_i = (\boldsymbol{\Phi}_i^T\boldsymbol{A}\boldsymbol{E})/(\boldsymbol{\Phi}_i^T\boldsymbol{A}\boldsymbol{\Phi}_i)$. Define q_i as the response of the model described as follows:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2q_i = -\ddot{u}_g \tag{6}$$

With these assumptions, the response of the structures is obtained through modal orthogonality and matrix transformation as follows [17]:

$$\boldsymbol{U}_s = \sum_{i=1}^{3n}[\boldsymbol{X}_iq_i + \boldsymbol{Y}_i\dot{q}_i] \tag{7}$$

where

$$\boldsymbol{X}_i = \frac{2}{a_i^2 + b_i^2}\left[(\xi_ip_i + \sqrt{1-\xi_i^2}w_i)\boldsymbol{\varphi}_{iR} + (\xi_iw_i - \sqrt{1-\xi_i^2}p_i)\boldsymbol{\varphi}_{iI}\right]\omega_i,$$

$$\boldsymbol{Y}_i = \frac{2}{a_i^2 + b_i^2}(p_i\boldsymbol{\varphi}_{iR} + w_i\boldsymbol{\varphi}_{iI}),$$

$$a_i = -2\eta_{iR}(\boldsymbol{\varphi}_{iR}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iR} - \boldsymbol{\varphi}_{iI}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iI}) - 4\eta_{iI}\boldsymbol{\varphi}_{iR}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iI} + \boldsymbol{\varphi}_{iR}^T\boldsymbol{C}_s\boldsymbol{\varphi}_{iR} - \boldsymbol{\varphi}_{iI}^T\boldsymbol{C}_s\boldsymbol{\varphi}_{iI},$$

$$b_i = 2\eta_{iI}(\boldsymbol{\varphi}_{iR}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iR} - \boldsymbol{\varphi}_{iI}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iI}) - 4\eta_{iR}\boldsymbol{\varphi}_{iR}^T\boldsymbol{M}_s\boldsymbol{\varphi}_{iI} + 2\boldsymbol{\varphi}_{iR}^T\boldsymbol{C}_s\boldsymbol{\varphi}_{iI},$$

$$p_i = a_ic_i + b_id_i, \; w_i = b_ic_i - a_id_i,$$
$$c_i = \boldsymbol{\varphi}_{iR}^T\boldsymbol{M}_s\boldsymbol{E}_s, \; d_i = \boldsymbol{\varphi}_{iI}^T\boldsymbol{M}_s\boldsymbol{E}_s.$$

As shown in the above derivation, the seismic response calculation of non-proportional damping structures is described by Eq. (7), which is based on the state equation and complex mode theory. The result is accurate without approximation.