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Mean crack spacing modelling for RC tension elements

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ABSTRACT

An approach for predicting the crack spacing of reinforced concrete tension elements is presented in this paper. Two well established methods, the mean strain and the stress transfer approaches (partial interaction) are combined through a newly proposed strain compliance principle. Through the assumption of a linear strain shape function governing the reinforcement strains in the stabilized cracking stage, the method enables the prediction of the mean crack spacing from the stress transfer approach. The strain compliance principle establishes the equality of the mean strains, estimated for a given loading level by a mean strain approach and the stress transfer approaches. The proposed approach requires a single data point, represented by a reinforcement ratio and bar diameter, that is denoted as a reference element and is further used to obtain the distance between cracks for any other tensile concrete element. A free-of-shrinkage tension stiffening law was employed for the evaluation of the mean reinforcement strains. A thorough investigation of this approach together with results for varying concrete compressive strengths, reinforcement ratio and bar diameters is presented. The new method was shown to give reasonably accurate results and most importantly, proposes a new way of investigating the cracking behavior of concrete elements.

1. Introduction

Cracking of concrete is among the most complex aspects in the analysis of Reinforced Concrete (RC) structures. Commonly the errors in crack width predictions can reach up to several hundred percent [1]. Such discrepancies are related to a number of physical and mechanical aspects in the behavior of RC structures, such as different mechanical properties of concrete and reinforcement bars, creep and shrinkage of concrete, bond-slip action, tension-stiffening and tension-softening effects, stochastic nature of the crack formation.

Cracking of RC structures has been studied since the beginning of XX century. A number of investigations may be summarized into two main groups:

(1) Empirical and semi-empirical models.
(2) Models, based on bond action and the stress transfer mechanism.

The first group represents the majority of early investigations, which were based on a large number of experimental data [2]. An attempt to summarize the experimental work done up to 1970s, was made by Gergely and Lutz [3]. Using the experimental data collected from available literature [4–8], Gergely and Lutz [3] performed a statistical analysis which resulted in the following best-fit equation for estimating the crack width:

$$w_{\max} = 0.091\sqrt[3]{t_b \cdot A} \cdot r(f_s - 5) \quad (1)$$

where $w_{\max}$ is the maximum crack width (mm); t_b is the bottom cover measured from the center of the lowest bar (mm); A is the average effective concrete area around a single bar; R is the distance from the neutral axis to the tension face and to the reinforcing steel centroid and f_s is the reinforcement stress (MPa).

Eq. (1) was introduced into the early version of ACI 318 design code [9] and is still often used for cracking analysis. An alternative approach to calculate the crack width in RC members was developed by Broms [8]. From the analysis of 37 tensile and 10 flexural members, the single most important parameter controlling the crack spacing was determined to be the concrete cover:

$$s_{rm} = 2t \quad (2)$$

where s_{rm} is the mean crack spacing and t is the distance from the center of the bar to the nearest surface. Broms [9] suggested to neglect the concrete strain between cracks and to calculate the crack width using the following equation:

$$w = 2t \cdot \varepsilon_{sm} \quad (3)$$

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where ε_{sm} is the average reinforcement strain.

The concrete cover approach was also used in other cracking models [10,11]. By analyzing results from a number of experimental studies [6,11,13–15], Beeby [16] also arrived to the conclusion that concrete cover is a far more important parameter in defining the cracking behavior than the ratio $\text{Ø}/\rho$ (where Ø is the bar diameter and ρ is the reinforcement ratio). The latter conclusion was also supported by the recent experimental results of RC tensile prisms reinforced with multiple bars [17].

The ratio $\text{Ø}/\rho$ is derived from the second group of cracking models, that are based on the bond theory. This approach was first developed by Saliger [18] and later was used in a number of studies and design codes [19–21]. According to these models, a crack may form after a certain fraction of the load is transmitted from the reinforcement to the concrete via bond action. Moving away from the cracked section, the reinforcement transfers stresses to the concrete until it reaches the tensile strength of concrete and, thus, a new crack may form. The distance, required to develop the tensile strength of concrete is often called the transfer length l_{tr} and is calculated from the equilibrium of the concrete cracking force and force transmitted by the bond action:

$$A_c f_{ct} = n\pi \text{Ø}_s \int_0^{l_{tr}} \tau(x)dx \tag{4}$$

where A_c is the tensile area of concrete; f_{ct} is the tensile strength of concrete; n is the number of bars $\tau(x)$ is the bond stress distribution function.

It may be understood, that a new crack may form only at the distance exceeding the transfer length l_{tr}. If the distance between two cracks is larger than $2l_{tr}$, a new intermediate crack will form, otherwise the length will not be sufficient to develop the tensile stresses greater than the tensile strength of the concrete. Therefore, the distance between cracks s_{rm} falls into a specific interval $l_{tr} \le s_{rm} \le 2l_{tr}$.

In the classical approaches, a constant bond stress distribution between cracks is assumed [12,18,22]. Such a simplification leads to a straightforward integration of the right part of Eq. (4), giving the following expression of transfer length:

$$l_{tr} = \frac{A_c f_{ct}}{n\pi \text{Ø}_s \tau_b} = \frac{1}{4}\frac{f_{ct}}{\tau_{b,\max}}\frac{\text{Ø}_s}{\rho} \tag{5}$$

The crack width is then calculated as a difference of displacements between the reinforcement and the concrete along the distance between two cracks:

$$w = s_{rm}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{6}$$

where ε_{sm} and ε_{cm} are the average strain of reinforcement and concrete between cracks, respectively.

To solve the equilibrium Eq. (4) and obtain the transfer length, more sophisticated approaches were developed, relating bond stresses to slip [23–25] or by assuming a certain shape function for the bond stress distribution [26,27]. Such approaches are also called *the stress transfer* or *partial interaction*, and are based on the solution of the second order differential equation, relating bond stresses and interfacial slip between the reinforcement and the surrounding concrete:

$$\frac{d^2 s}{dx^2} = \tau \frac{4(1+\alpha\rho)}{E_s \text{Ø}_s} \tag{7}$$

where s is the slip; α is the ratio of elastic moduli of reinforcement and concrete.

In such approaches, the cohesive stresses transferred by concrete through the crack may also be taken into account [28]. However, the results of cracking analysis using stress transfer approach is principally governed by the assumed bond-slip relationship [24]. As the bond-slip behavior of reinforcement may vary considerably in experiments, depending on the structural element and test type (pull-out, direct tension or bending specimen), the distribution of the reinforcement in the section and concrete cover, the obtained crack width values may be accurate only for particular elements [29].

Despite the extensive studies, started in the early sixties [23] and carried out until now, there are no bond-slip laws that would ensure adequate cracking analysis results for a wide range of geometrical, loading and material characteristics of elements.

The presented two main theories often deliver controversial crack width analysis results, that highly depend on the geometry of the specimen and the type of reinforcement [16]. A number of variations of the two main approaches were also proposed, that mostly alter the empirical coefficient values [30]. In a critical review on crack control of RC structures, A. Windisch [31] discusses the shortcomings of the international codes regarding their inability at making a distinction between the primary and secondary cracks. Nevertheless, errors in crack width predictions are still scattered and yield unreliable results in cracking analysis of RC structures.

The present paper aims to propose a new methodology for carrying out cracking analysis of tensile RC ties at the stabilized cracking stage. The authors combine the main principles of the stress transfer approaches with the average deformation behavior (mean strain approach) of the tension element. The former approach governs the strain distribution of reinforcement between cracks, whereas the latter takes advantage of the knowledge of the mean strain of the element. The introduced technique enables the derivation of mean crack spacing models that reflect the main mechanical principles of reinforcement and concrete interaction and are compatible with the deformation behavior of the member.

2. Behavior of RC tie

A RC element shown in Fig. 1 is used to illustrate cracking, deformation and bond behavior of RC structures. Such elements are often chosen due to simplicity and reasonably good representation of the distribution of internal forces and strains in the tensile zone of RC structures [32].

At the initial loading stage (*OA*) the deformation behavior of the tensile member is almost linear elastic, as shown in Fig. 1a. The composite action and compatibility of reinforcement and concrete strains are attained in the element with slip occurring only in the small regions at the loaded ends. Bond stresses are directly related to slip and, therefore, develop in the regions with non-zero slippage. Bond stresses increase together with the value of slip, whereas at the ends of the segments between cracks bond stresses diminish until they reach zero at the location of cracks. This effect can be explained by the presence of localized concrete damage near the crack plane, which significantly reduces the bond action [33].

With increasing load, strains in reinforcement as well as in concrete grow until a certain limit of concrete cracking is reached (Point *A* in the load-average strain diagram of Fig. 1a). The first crack appears in the section where stresses in concrete transferred through bond action reach its tensile strength. This causes an immediate redistribution of stresses in the cracked section: concrete stresses and strains drop to zero at the location of the crack, thus the entire tensile force is transferred only through the reinforcement bar (Fig. 1b). Further away from the crack, part of tensile force is transferred to concrete through the bond action. Stresses and strains in the concrete increase over distance until the tensile strength of concrete is reached and a new crack may develop. The distance required to reach the tensile strength of concrete is often