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Post-buckling of functionally graded microplates under mechanical and thermal loads using isogeomertic analysis

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ABSTRACT

The present study uses the isogeometric analysis (IGA) to investigate the post-buckling behavior of functionally graded (FG) microplates subjected to mechanical and thermal loads. The modified a strain gradient theory with three length scale parameters is used to capture the size effect. The Reddy third-order shear deformation plate theory with the von Kámán nonlinearity (i.e., small strains and moderate rotations) is employed to describe the kinematics of the microplates. Material variations in the thickness direction of the plate are described using a rule of mixtures. In addition, material properties are assumed to be either temperature-dependent or temperature-independent. The governing equations are derived using the principle of virtual work, which are then discretized using the IGA approach, whereby a C^2-continuity requirement is fulfilled naturally and efficiently. To trace the post-buckling paths, Newton's iterative technique is utilized. Various parametric studies are conducted to examine the influences of material variations, size effects, thickness ratios, and boundary conditions on the post-buckling behavior of microplates.

1. Introduction

Functionally graded materials (FGMs) [1] are a new class of composite materials that have been employed in various engineering applications. The adoption of FGMs can eliminate interface problems and alleviate thermal stress concentrations in structural components, which are the major concerns with conventional laminated composites. These desirable features of FGMs are naturally obtained since their materials vary smoothly and continuously as functions of position along certain spatial direction. In general, a FGM is made from two different material constituents, for example, ceramic and metal phases. The ceramic constituent is considered as a high temperature barrier, thanks to its low thermal conductivity, whereas the metal counterpart is more ductile to prevent fracture due to thermal stresses. In addition to the linear analysis of FG plates [2], a large number of studies have been devoted to investigate their post-buckling responses [3–18]. A comprehensive review of plate theories and techniques used to analyse FG plates subjected to mechanical and thermal loads can be found in [2,19,20].

In recent years, beams and plates whose dimensions are in microns and nanometers have been widely employed in advanced devices, such as microelectromechanical systems (MEMS) and nano electromechanical systems (NEMS). With the distinguishing features, the FGMs were also employed in such applications [21,22]. In fact, the behavior of small scale structures is considerably influenced by material size effects, which can only be captured using non-classical elasticity theories with length scale parameters. In the literature, nonlocal elasticity proposed by Eringen [23], Modified Couple stress Theory (MCT) of Yang et al. [24] and Modified Strain gradient Theory (MST) of Lam et al. [25] are widely used to investigate the behavior of micro/nano structures. Recent studies on the developments and applications of nonlocal theory to nanobeams and nanopaltes could be found in the works of Li et al. [26–29], Nguyen et al. [30], and Phung-Van et al. [31]. Based on the MCT, numerous studies have been also carried out to investigate the behavior of micro beams and plates [32–45]. For nonlinear analysis in accordance with the MCT, Reddy and Kim [41] developed a model based on a general third-order shear deformation plate theory, which can be specialized to classical,

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first-order shear deformation, the Reddy third-order plate theories and accounts for both von Kármán nonlinearity and FGMs. Ke et al. [46,47] also investigated the nonlinear free vibration of FG annular microplates and microbeams by adopting MCT. Based on the classical and first-order theory, Reddy et al. [48] presented finite element models of microstructure-dependent nonlinear theories for axisymmetric bending of FG circular plates, which account for the through-thickness power-law variation of a two-constituent materia, the von Kármán nonlinearity, and the couple stress effects. Compared to the MCT, the MST is more extensive and general in dealing with micro structures. In addition, the MST is less complicated in terms of mathematical formulations than the classical strain gradient theory [49], which can have more length scales. Therefore, various studies has been published in the literature to discuss the size effect in micro structures based on the MST. Wang et al. [50] and Ashoori Movassagh and Mahmoodi [51] developed the Navier solutions for bending, vibration, and buckling responses of isotropic microplates. Sahmani and Ansari [52] also employed the analytical approach to examine the free vibration behavior of FG microplates using higher-order plate theories. Kirchhoff's plate theory was adopted by Li et al. [53] to predict the bending response of bi-layer microplates. Ansari et al. [54] and Zhang et al. [55] derived the displacements, frequencies and buckling loads of annular/circular FG microplates. Zhang et al. [56] also developed an analytical model using the refined plate theory to investigate the response of FG microplates resting on an elastic foundation. The buckling and free vibration responses of FG microplates were also addressed in the study of Mirsalehi et al. [57] by using the finite strip method. Hosseini et al. [58] employed an analytical approach to study the buckling responses of an orthotropic multi-microplate system embedded in an elastic medium. Thai et al. [59] also carried out a comprehensive investigation on linear bending, free vibration and buckling responses of FG microplates by using IGA approach. In addition, the behavior of microplates under thermal loads was also examined based on the MST in some studies recently. For example, Ansari et al. [60] presented an investigation on the linear thermal buckling of FG microplates based on the first-order plate theory. Shenas and Malekzadeh [61] discussed the influence of thermal environment on the free vibration behavior of quadrilateral FG microplates by using the Chebyshev-Ritz method. Emami and Alibeigloo [62] also adopted the first-order plate theory and analytical approach to study the thermoelastic damping behavior of FG microplates, in which the coupled thermoelasticity was investigated together with the size effect.

A review of the literature [22] shows that there are very few studies on the post-buckling behavior of microplates based on the MST so far. From an analytical point of view, the post-buckling behavior of plate structures could be investigated using different approaches, such as analytical, semi-analytical, and numerical methods. The analytical approach is only suitable for the problems with simple geometries and boundary conditions, whereas other numerical methods encounter the difficulty in fulfilling the high-order continuity requirement of the interpolation functions, for example, when the first- or high-order shear deformation theory is considered. The IGA approach [63] which is known as advanced computational techniques have been extensively employed to deal with various problems in many fields of computational mechanics. The core idea of IGA is to employ the predominant technology used in Computer-Aided Design (CAD) as geometry discretization technique and discretization tool for analysis. Non-uniform rational B-splines functions inheriting the advanced features in geometrical modelling allow the IGA approach not only to be able to generate highly continuous interpolations but also present exactly the geometries of arbitrary conical shapes, hence the error in geometrical modelling could be alleviated. For shell and plate problems in particular, the smoothness of basis functions obtained from IGA approach allows the construction of plate/shell element less complicated compared to other techniques [64]. In addition, k-refinement which is unique to IGA presents a robust and systematic technique to elevate the continuity of interpolation functions efficiently and naturally.

In this study, the post-buckling behavior of FG microplates under mechanical or thermal loads is investigated using IGA-based numerical model and the MST. The shear deformation effect is also investigated using the Reddy third-order shear deformation theory [65]. The von Kámán nonlinear strains and temperature dependent properties are accounted for. The principle of virtual work is utilized to derive the weak form equation of the problem. Based on the IGA technique, the C^2-continuity of the interpolation functions is met naturally and efficiently. The post-buckling paths are traced using Newton's iterative procedure with small initial imperfection. Verification exercises are performed to present the accuracy of the present approach. In addition, various parametric studies are carried out to determine the influences of the power-law indices, size effects, thickness ratios, and boundary conditions on the post-buckling response of FG microplates.

2. Material properties of FGMs

For a FG rectangular plate whose geometry and coordinates are depicted in Fig. 1, the material properties such as Young's modulus $E(z)$, thermal conductivity $\kappa(z)$, and thermal expansion $\alpha(z)$ vary throughout the thickness h according to the rule of mixture as [3]

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + P_m \tag{1}$$

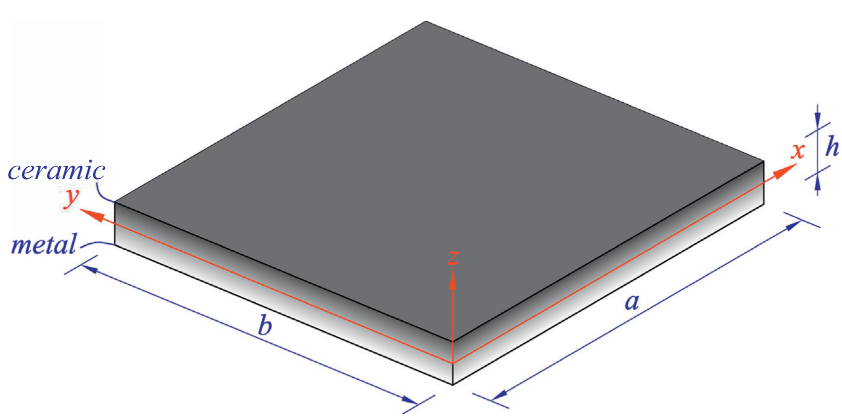

Fig. 1. Configuration of a rectangular FG plate.