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Effect of linear transformation on nonlinear behavior of continuous prestressed beams with external FRP cables

Tiejiong Lou [a,b,*], Sergio M.R. Lopes [b], Adelino V. Lopes [b]

[a] Hubei Key Laboratory of Roadway Bridge & Structure Engineering, Wuhan University of Technology, Wuhan 430070, China
[b] Department of Civil Engineering, University of Coimbra, Coimbra 3030-788, Portugal

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ABSTRACT

In continuous prestressed members, linear transformation of the cable line is an interesting topic that has not yet been adequately addressed. This paper presents a numerical study on the flexural response of two-span continuous externally fiber reinforced polymer (FRP) prestressed concrete beams having various linearly transformed cable profiles, with emphasis on the redistribution of moments. The study is conducted by using a robust computational method that has been validated against experimental tests. The results confirm that the cable shift by linear transformation does not affect the basic performance at all stages of loading up to failure. The secondary moments are shown to vary linearly with the cable shift. The development of moment redistribution over the center support is significantly affected by linear transformation. An upward cable shift leads to a decrease in moment redistribution at ultimate. The results indicate that the effect of linear transformation on moment redistribution is neglected or incorrectly included in various design codes.

1. Introduction

External post-tensioning has been widely used in the strengthening of existing concrete bridges as well as in the construction of various new structures [1,2]. Since external cables are placed at the exterior of a structural member, a strict anticorrosive requirement of the cables is essential. Fiber reinforced polymer (FRP) composites are noncorrosive materials with high tensile strength, and are recognized as a promising alternative to conventional prestressing steel [3]. The composite cables may be made of glass [4], aramid [5], carbon [6] and basalt fibers [7]. Special anchorage systems for FRP cables are required in prestressing applications [8,9].

The redundant constraints in continuous prestressed members may cause different structural behavior from that of simply supported members and may introduce new aspects of behavior such as secondary moments. In an early work by Lin and Thornton [10], a simple and conservative method was proposed to reveal the secondary moments in the inelastic range but no exact solution was given. By using an analytical model based on the moment-curvature relationship, Wyche et al. [11] studied the secondary moments in continuous prestressed concrete beams at various stages of loading up to the ultimate. Lou et al. [12] proposed a refined method for computing secondary moments throughout the loading history and found that the secondary moment increases linearly with the tendon stress. It is well known that bending moments in continuous beams will be redistributed during the inelastic range of loading. Moment redistribution in steel prestressed members has been broadly addressed by different investigators [13–18]. Unlike steel which exhibits obvious ductility, FRP materials are linearly elastic materials without yielding. Consequently, concrete beams with FRP reinforcement may redistribute moments in a different way when compared to those with steel reinforcement. A few studies have recently been undertaken to explore moment redistribution in continuous prestressed concrete beams with FRP tendons [19–21].

One of the interesting topics related to prestressed continuous members is linear transformation (cable movement at inner supports without altering the intrinsic shape of the cable line). Lin and Burns [22] pointed out that the ultimate load of a continuous prestressed beam would not be influenced by linear transformation, but no theoretical or experimental proofs were provided. Based on experimental observations from 3 symmetrically and 2 unsymmetrically loaded externally prestressed beams with linearly transformed cable profiles, Aravinthan et al. [23] concluded that the flexural behavior of continuous beams, both at the elastic and ultimate limit states, is independent of the layout of the cables.

* Corresponding author at: Hubei Key Laboratory of Roadway Bridge & Structure Engineering, Wuhan University of Technology, Wuhan 430070, China.

E-mail address: loutiejiong@dec.uc.pt (T. Lou).

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Similar observation was reported in a numerical study by Lou et al. [12].

Although some efforts have been made, the knowledge about linear transformation of the cable profile has not yet been fully established. For example, a cable shift by linear transformation is expected to cause a significant change of secondary moments, which may result in different moment redistribution behavior. However, there is no report regarding the effect of linear transformation on moment redistribution in prestressed continuous beams. In this paper, nonlinear behavior of continuous externally FRP prestressed beams having various linearly transformed cable profiles is investigated, emphasizing on redistribution of bending moments.

2. Nonlinear model

The present numerical investigation is conducted by using a finite element analysis (FEA) model developed based on the following basic assumptions: a plane section remains plane after deformations; there is perfect bond between nonprestressed steel and concrete; and shear deformations are negligible. In addition, the analysis assumes that friction between cables and deviators is negligible, as evidenced by some real bridges with external cables [24]. This simplification was commonly accepted in previous studies on external prestressing systems, but it may lead to an overestimation of the cable stress if frictional effects are not minimized. A method for assessing friction losses at deviators can be found in Au et al. [25].

The material stress-strain curves are shown schematically in Fig. 1. The stress-strain equation for concrete in compression is that recommended in EC2 [26] as shown in Fig. 1(a), where $\eta = \varepsilon_c/\varepsilon_{c0}$; σ_c and ε_c are the concrete stress and strain, respectively; $\varepsilon_{c0}(‰) = 0.7 f_{cm}^{0.31}$; $f_{cm} = f_{ck} + 8$ in which f_{ck} is the concrete characteristic cylinder compressive strength, in MPa; $k = 1.05 E_c \varepsilon_{c0}/f_{cm}$; and $E_c = 22(f_{cm}/10)^{0.3}$, in GPa. The concrete in tension is assumed to be linearly elastic prior to cracking and linear strain-softening after cracking, as shown in Fig. 1(b). The concrete tensile strain at the end of strain-softening is taken as $10\varepsilon_{cr}$, where ε_{cr} is the cracking strain. FRP cables are linearly elastic up to rupture as shown in Fig. 1(c). The nonprestressed steel is assumed to be elasto-perfectly plastic as shown in Fig. 1(d).

Consider a two-node beam element in the local coordinate system (x, y), as shown in Fig. 2(a). Each node has three degrees of freedom, i.e., the axial displacement u, transverse displacement v and rotation θ. Assume that the axial and transverse displacements are a linear function and a cubic polynomial, respectively. By applying the principle of virtual work, the element tangential equilibrium equations can be determined as follows [27]:

$$d\boldsymbol{R}^e = \boldsymbol{K}_t^e d\boldsymbol{r}^e = (\boldsymbol{K}_o^e + \boldsymbol{K}_g^e) d\boldsymbol{r}^e \tag{1}$$

$$\boldsymbol{K}_o^e = \int_V \boldsymbol{B}^T E_t \boldsymbol{B} dV, \quad \boldsymbol{K}_g^e = \int_V \sigma \boldsymbol{J}^T \boldsymbol{J} dV \tag{2}$$

$$\boldsymbol{B} = [\boldsymbol{N}_1' \quad -\boldsymbol{N}_2'' y], \quad \boldsymbol{J} = [0 \quad 0 \quad N_2'] \tag{3}$$

where $\boldsymbol{R^e}$ is the element equivalent nodal loads written as $\{N_i, N_j, V_i, V_j, M_i, M_j\}^T$; $\boldsymbol{r^e}$ is the element nodal displacements written as $\{u_i, u_j, v_i, v_j, \theta_i, \theta_j\}^T$; K_t^e is the element tangential stiffness matrix, which is composed of the small-displacement stiffness matrix K_o^e and geometric stiffness matrix K_g^e; E_t represents the tangential modulus; σ represents the stress; $\boldsymbol{N}_1 = [N_1, \ N_2]$; $\boldsymbol{N}_2 = [N_3, \ N_4, \ N_5, \ N_6]$; $N_1 = 1 - \xi$; $N_2 = \xi$; $N_3 = 1 - 3\xi^2 + 2\xi^3$; $N_4 = 3\xi^2 - 2\xi^3$; $N_5 = l(\xi - 2\xi^2 + \xi^3)$; $N_6 = l(-\xi^2 + \xi^3)$; $\xi = x/l$ in which l is the length of the element. A superimposed prime in Eq. (3) corresponds to the differentiation with respect to x. After assembling the structure equilibrium equations and imposing appropriate boundary conditions, an incremental method combined with the Newton-Raphson iterative scheme is employed for the numerical solution.

The external cable can be considered as an assemblage of cable segments each of which spans a beam element, as shown in Fig. 2(b), in which e_i and e_j are the tendon eccentricities at nodes i and j, respectively. For external tendon systems, the tendon eccentricities will change with member deformations, except at anchorage and deviator points. At each step, the location of each cable segment can be determined according to the current coordinates of the anchorage and deviator points. The eccentricities e_i and e_j are then updated in terms of the coordinates of the cable segment joints (pi and pj) and the beam element nodes (i and j), thus allowing the second-order effects to be considered in the numerical procedure. Meanwhile, the tendon strain increment is calculated by the elongation of the entire cable, and then the tendon strain, stress and force (N_p) can be obtained. When the tendon eccentricities and force are determined, the equivalent nodal loads resulting from external prestressing can be easily computed, as can be seen in Fig. 2(b). Details about numerical modeling of external prestressing cables have been reported elsewhere [27,28].

3. Model validation

Aravinthan et al. [23] tested a series of prestressed concrete beams with highly eccentric external cables to study their flexural behavior. Three of the beams are analyzed herein for validating the proposed model. These beams were A-1, B-1 and C-1 with section and structure details shown in Fig. 3. The major variable was the layout of the external cable, i.e., three different linearly transformed cable profiles were used. The beams were 10.4 m long, continuous over two equal spans of 5.0 m each. The rectangular cross section was 400 mm wide and 150 mm deep. Each span was subjected to two concentrated loads with a spacing of 1.25 m. The beams were pre-tensioned with 4 internal bonded steel cables with an area of 51.61 mm^2 each and also post-tensioned with one external steel cable with an area of 69.68 mm^2. The external cable went through the concrete beam at certain locations through embedded steel pipes. Vertical struts with various lengths were placed along the beam to achieve desired cable profiles. The ultimate strength and elastic modulus of the cables were 1722 MPa and 196.2 GPa,

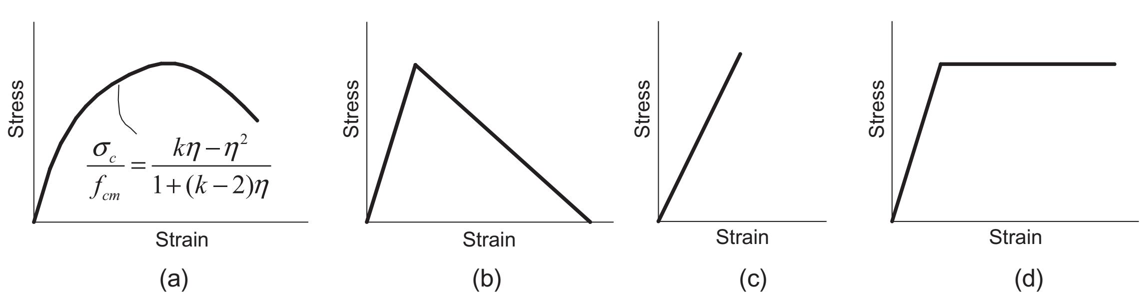

Fig. 1. Stress-strain diagrams of materials. (a) Concrete in compression; (b) concrete in tension; (c) FRP cables; (d) nonprestressed steel.