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Second-order torsional warping theory considering the secondary torsion-moment deformation-effect

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ABSTRACT

In this paper, the influence of the variable axial force and of the Secondary Torsion-Moment Deformation-Effect (STMDE) on the deformations of beams due to torsional warping is investigated. The investigation is based on the second-order torsional warping theory of doubly symmetric beams with thin-walled open or closed cross-sections. The effect of the axial force on the torsional stiffness of thin-walled beams is considered according to the second-order torsional warping theory. The solutions of the underlying differential equations are used for setting up the relations, needed for application of the transfer matrix method. They are derived, considering both static and dynamic action. This enables stablishing the local element matrix of the twisted beam in the framework of the Finite Element Method (FEM). The numerical investigation comprises static and modal analyses of thin-walled beams with I cross-sections and rectangular hollow cross-sections. The results are compared with results obtained by the FEM, using solid and beam elements available in standard software.

1. Introduction

The effect of non-uniform torsion must be considered in structural analysis of thin-walled beams with open as well as closed cross-sections. The maximum axial stress caused by the bimoment occurs at the points of action of external torques (except for free ends of beams) and at cross-sections of restrained warping (for example clamped cross-sections). A broad comprehensive overview of the literature dealing with the issue of non-uniform torsion can be found, for example, in [1,2]. Recent research results have shown that for non-uniform torsion of beams with closed cross-sections the influence of the Secondary Torsion-Moment Deformation-Effect (STMDE) is particularly significant.

Beam structures are frequently exposed to dynamic loads. Commercial FEM codes enable modal and transient analysis by 3D finite beam elements without and with consideration of warping [3–5]. For torsion, very often an improved Saint-Venant theory is used and special mass matrices are considered. In general, the bicurvature is chosen as an additional warping degree of freedom, and the STMDE is not considered (Ref. [5] is an exception). The beam element in [4] can be used with a lumped or a consistent mass matrix. The consistent mass matrix includes warping effects, but does not include the effect of shear deformations. For standard beam elements, the consistent mass matrix is based on Ref. [6], with the exception of additional terms arising from the warping constant I_ω. For the warping element, lumped masses for the warping degree of freedom (bicurvature) are defined in [7]. As stated in [4], for solid and closed thin-walled sections, standard finite beam elements can be used without significant error. However, for open thin-walled sections, warping finite beam elements should be used. In [5], however, the warping finite beam element is recommended only for use for open thin-walled section beams. In [8], a boundary element method is developed for the non-uniform torsional vibration problem of doubly symmetric constant cross-sections, accounting for non-uniform warping and secondary torsional shear deformation-effects. Dynamic analysis of 3D beam elements, restrained at their edges and subjected to arbitrarily distributed dynamic loading is described in [9]. In [10], an elastic

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non-uniform torsion analysis of simply or multiply connected cylindrical bars with arbitrary cross-sections accounts for the effect of geometric non-linearity in the framework of the boundary-element method. In [11], the effect of rotary and warping inertia is considered. Nonlinear torsional vibrations of thin-walled beams, exhibiting primary and secondary warping, are investigated in [12]. A solution for the vibrations of Timoshenko beams by the isogeometric approach is presented in [13]. Warping effects, however, are not considered. In [14], geometrically nonlinear free and forced vibrations of beams with non-symmetrical cross-sections are investigated by the Saint-Venant theory of torsion. Axial-torsional vibrations of rotating pretwisted thin-walled composite box beams, exhibiting primary and secondary warping, are investigated in [15]. A formulation of a 3D beam element for computation of transversal and warping eigenmodes is presented in [16].

In [17], a new 3D finite element for geometrically nonlinear analysis of beams, made of Functionally Graded Material (FGM) with transversally varying material properties, is presented. The warping displacements are accurately predicted.

In [1], the influence of torsional warping of open and closed cross-sections of twisted beams, made of materials with constant material properties, on their eigenvibrations is investigated, considering the secondary deformations due to the angle of twist. Since the bicurvature cannot be used in the constraint equations, see. e.g. [4], it was logical to consider the part of the first derivative of the angle of twist, caused by the bimoment, as the warping degree of freedom [18] also for modal analysis. The results from modal analysis, concerning non-uniform and uniform torsion of beams with open cross-sections, have shown large differences of the eigenfrequencies. This has corroborated the well-known fact that warping must be taken into account also for modal analysis of beams with open cross-sections, subjected to torsion. It was also shown that the STMDE does not play a significant role in torsion of beams with open cross-sections. On the other hand, the torsional eigenfrequencies, obtained in case of consideration of STMDE, are very close to the ones obtained by 3D solid finite elements. In contrast to open cross-sections, the influence of warping (with or without STMDE) on the non-uniform torsional eigenfrequencies of beams with rectangular hollow cross-sections is not significant. The best agreement of results obtained by solid finite elements and by the method proposed in [1] (both for the Saint-Venant and the warping beam solutions) is obviously achieved for the first torsional eigenfrequency. For the higher modes, the difference between corresponding results increases especially for short beams. Some higher torsional eigenmodes, calculated by means of solid finite elements, contain deformations of the side walls of the beams. This effect cannot be considered in a straightforward manner by finite beam elements with restrained and unrestrained warping. As shown in [19], all eigenfrequencies calculated by solid finite elements agree very well with results obtained by experimental measurements.

Other very recent aspects in the area of numerical solutions of non-uniform torsion are treated in [20–24]. Finally, in [2], a boundary element solution is developed for dynamic analysis, considering warping of beams with arbitrary cross-sections, including shear lag effects due to both flexure and torsion. High accuracy of the results in comparison to the ones obtained by solid finite element solution is obtained. However, in the solid model, the distortion effect of the cross-section was restrained.

A common feature of the above cited articles is disregard of the effect of the variable axial force on torsional warping.

In this paper, the work reported in [1] is extended to uniform and non-uniform torsional analysis of beams with a variable axial force. In Chapter 2, the differential equations of beams with such an axial force are formulated for Saint-Venant and non-uniform torsional deformations, including inertial line moments. In non-uniform torsion, the part of the bicurvature, caused by the bimoment, is taken into account as the warping degree of freedom, and the STMDE is also considered. A general semi-analytical solution of the differential equation is presented in Chapter 3, the transfer matrix relation is established in Chapter 4, from which the finite element equations for beam elements with two nodes are derived in Chapter 5. Omitting the external load, the FEM equation for the torsional natural free vibrations is obtained. The numerical investigation in Chapter 6 deals with torsional modal and elastostatic analysis of thin-walled beams with I cross-sections and rectangular hollow cross-sections. The obtained results are compared with the ones from commercial FEM codes. The effect of the axial force is evaluated. A final assessment of the proposed method is contained in the conclusions. Some of the mathematical details are explained in the Appendix.

The main novelties of the present paper are:

(1) consideration of a variable axial force and of the STMDE in the differential equation for non-uniform torsion of thin-walled beams with open and closed cross-sections according to the theory of second-order torsional warping;
(2) formulation of the equations needed for the transfer matrix method and the FEM for elastostatic and modal analysis of non-uniformly twisted beams according to this theory.

2. Differential equation of the torsional deformations of beams with variable axial forces

According to the theory of second-order torsional warping, the axial forces affect the torsional stiffness GI_T, where G is the shear modulus and I_T is the torsion constant. Basically, compression results in a decrease and traction in an increase of the torsional stiffness GI_T of the beam. This situation may be considered by an additional stiffness $N i_p^2$ (e.g. [25]) for doubly symmetric cross-sections, where N is the known axial force, acting at the center point of the cross-section, and $i_p = \sqrt{I_P/A}$ denotes the radius of gyration and I_P is the polar moment of area. In case of a variable axial force $N^{II}(x) = N(x)$, the corresponding variable torsional stiffness is obtained as $GI_T^*(x) = GI_T + N^{II}(x)\, i_p^2$, where the term $N^{II}(x)\, i_p^2$ denotes the so-called geometric stiffness. Representing a load, the axial force $N(x)$ appears in the respective term of the differential equation for the displacement in the longitudinal direction. The variable axial force $N^{II}(x)$ appears in the homogeneous part of the differential equation for the angle of twist. The variation of the known axial force $N^{II}(x)$ accounts for the stiffening or softening of the torsional stiffness in the framework of the second-order torsional warping theory. For doubly symmetric cross-sections, the torsional deformations are decoupled from the bending deformations and the longitudinal deformations. For this case, the differential equation for the angle of twist will be established in this Chapter.

Fig. 1 refers to the second-order torsional warping theory. It shows the axial force $N^{II}(x)$, the torsional moment $M_T(x)$ as the sum of the primary torsional moment, $M_{Tp}(x)$, and the secondary torsional moment, $M_{Ts}(x)$, and the bimoment $M_\omega(x)$. Fig. 1 also

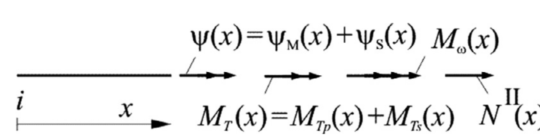

Fig. 1. Second-order torsional warping theory: axial force, torsional moments and angles of twist.