Engineering Structures 142 (2017) 128–136

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Topological design of structures under dynamic periodic loads

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ARTICLE INFO

Article history:
Received 10 November 2016
Revised 28 March 2017
Accepted 29 March 2017

Keywords:
Topology optimization
Dynamic compliance
Bi-directional evolutionary structural optimization (BESO)
Solid-void design

ABSTRACT

Most of forces acted on real structures are dynamic and topology optimization of dynamic structures has aroused wide attention over the past years. Due to the complexity of dynamic behavior, achieving clear 0/1 optimal topology of dynamic structures is still challenging. This paper aims to develop a topology optimization algorithm of dynamic structures under periodic loads based on the bi-directional evolutionary structural optimization (BESO) method. To minimize the dynamic compliance under the single or multiple excitation frequencies, four typical topology optimization problems are proposed for different scenarios. To solve the defined topology optimization problems, sensitivity analysis with regard to the variation of design variables is conducted for iteratively updating the structural topology. Since BESO uses discrete design variables, the resulting solid-void solutions show unambiguous topologies of dynamic structures. Various 2D and 3D numerical examples are given to demonstrate the capability of the proposed method for obtaining optimal designs of dynamic structures under periodic loads.

1. Introduction

Topology optimization of continuum structures aims to find the best distribution of materials within a given design domain. Since the landmark paper of Bendsøe and Kikuchi [1] in 1988, many topology optimization techniques, such as the solid isotropic material with penalization (SIMP) [2,3], level set [4,5], the evolutionary structural optimization (ESO) [6,7] and bi-directional ESO (BESO) [8,9], have been developed and successfully demonstrated on the minimum compliance of static continuum structures.

Since most of forces acted on real structures are dynamic, topology optimization for dynamic structures has attracted wide attention over the past decades. For free vibrating structures, frequency optimization aims to drive natural frequency away from unfavorable frequency range and has been investigated by the homogenization method [10,11] and the SIMP method [12–14] by relaxing discrete design variables to continuous ones. ESO was recognized as a hard-kill method, which directly removes elements from the ground structure [15]. The occurrence of the artificial localized modes can be avoided due to removing elements. Nevertheless, such a hard-kill algorithm is often problematic for frequency optimization since the sensitivities of removing elements cannot be calculated and are wrongly assumed to be zero [16,17]. Therefore, Huang et al. [18] developed a “soft-kill” BESO method for topology optimization of structures by maximizing natural frequency, where the sensitivities of “soft” elements were accurately calculated. The study successfully obtained convergent solutions with clear 0/1 optimal topologies for frequency optimization problems.

Topology optimization of dynamic structures for forced vibration is of great importance in many engineering fields, e.g., constructional engineering and mechanical engineering. Similar to forced vibration analysis, topology optimization of dynamic structures can be conducted in the time domain or the frequency domain. Topology optimization of dynamic structures in the time domain aims to maximize the instant stiffness or strain energy of the structure within the specified time [19–22]. Thus, the selection of the terminate time is critical. Since the transfer of applied loads to supports needs some time, topology optimization using a short termination time may result in an unrealistic design, e.g., with material distributed around the applied loads without any supports. However, a long terminate time leads to expensive computation burden for topology optimization [22].

Alternatively, dynamic response topology optimization can be conducted efficiently in the frequency domain for structures subject to periodic loads. Ma et al. [23] proposed the homogenization method to tackle topology optimization problems for optimizing dynamic response by minimizing the dynamic compliance. Jog

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http://dx.doi.org/10.1016/j.engstruct.2017.03.067
0141-0296/

[25] optimized dynamic structures subject to the periodic loading with the local and global definition of dynamic compliances. Shu et al. [25] defined the dynamic response of connection points or surfaces as the objective function to suppress the vibration on these points or surfaces by using the level set method. Olhoff and Du [26,27] studied topology optimization of structures under forced vibration by minimizing dynamic compliance using the SIMP method. Liu et al. [28,29] minimized the displacement amplitude at the specified location of structures under harmonic force excitations. Kang et al. [30,31] investigated the optimal distribution of damping material in vibrating structures. Zhang and Kang [32] developed the aggregated dynamic compliance in a given frequency range for topology optimization of piezoelectric layers in plates for the best vibration control performance.

Although topology optimization of structures under dynamic loads have been extensively investigated [23–32] based on finite element analysis, the discrete 0/1 design variables were normally relaxed with continuous ones. Thus, finite element models unavoidably contained artificial intermediate elements, whose design variables are between 0 and 1. Such intermediate elements may cause local resonance and further lead to a wrong search direction of optimization if uncorrected. Furthermore, the dynamic response of the resulting optimized designs may also be inaccurate due to the existence of intermediate elements. BESO directly uses discrete design variables, which has potential to avoid these problems. Hence, this paper aims to extend the BESO method to topology optimization of dynamic structures under forced vibration to eliminate the local vibration modes and achieve clear 0/1 optimized topologies. The paper is organized as follows: Section 2 briefly introduces finite element analysis for dynamic structures in the frequency domain. Section 3 presents various topology optimization problems for dynamic structures under periodic loads, then follows sensitivity analysis. Section 4 describes numerical implementation of the BESO procedure. Numerical examples and discussion are presented in Section 5 to demonstrate the effectiveness of the proposed method. Conclusions are drawn in Section 6.

2. Finite element analysis

When a continuum structure is discretized with finite elements, the equilibrium equation for dynamic response of the structure subject to a force $\mathbf{P}_t$ with the time varying is written as

$$\mathbf{M}\ddot{\mathbf{X}}_t + \mathbf{C}\dot{\mathbf{X}}_t + \mathbf{K}\mathbf{X}_t = \mathbf{P}_t \tag{1}$$

where $\ddot{\mathbf{X}}_t$, $\dot{\mathbf{X}}_t$ and $\mathbf{X}_t$ are the acceleration, velocity and displacement, respectively. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent the structural mass, damping and stiffness matrices, respectively. The structural mass matrix, $\mathbf{M}$, and stiffness matrix, $\mathbf{K}$, can be expressed by

$$\begin{cases} \mathbf{M} = \sum_{i=1}^{NE} \mathbf{m}_i \\ \mathbf{K} = \sum_{i=1}^{NE} \mathbf{k}_i \end{cases} \tag{2}$$

where NE is the total number of elements in the design domain. $\mathbf{m}_i$ and $\mathbf{k}_i$ are the elemental mass and stiffness matrices, which are calculated by

$$\begin{cases} \mathbf{m}_i = \int_{V_i} \rho_i \mathbf{N}^T \mathbf{N} dV \\ \mathbf{k}_i = \int_{V_i} \mathbf{B}^T \mathbf{D}_i \mathbf{B} dV \end{cases} \tag{3}$$

where V_i is the volume domain of the i-th element, ρ_i and $\mathbf{D}_i$ are the mass density and constitutive matrix, respectively. $\mathbf{N}$ and $\mathbf{B}$ represent elemental shape function and strain-displacement matrices.

It is assumed that Rayleigh damping model can be adopted and the damping matrix is simplified by a linear combination of $\mathbf{M}$ and $\mathbf{K}$ [33] as

$$\mathbf{C} = \beta_1 \mathbf{M} + \beta_2 \mathbf{K} \tag{4}$$

where β_1 and β_2 are the damping coefficients. They can be determined by

$$\beta_1 = \frac{2\omega_i\omega_j(\omega_j\xi_i - \omega_i\xi_j)}{\omega_j^2 - \omega_i^2}, \quad \beta_2 = \frac{2(\omega_j\xi_j - \omega_i\xi_i)}{\omega_j^2 - \omega_i^2}$$

where ζ_i and ζ_j are modal damping parameters corresponding to the two different natural frequencies ω_i and ω_j. ω_i donates the smallest natural frequency and $\omega_j > \omega_i$ is selected by the loading response. In this paper, both of the two modal damping parameters are assumed to be 0.01. ω_i and ω_j are the first two natural frequencies and the resulting damping matrix is positive definite.

A harmonic load at a given excitation frequency, $\mathbf{P}_t = \mathbf{P}e^{i\omega t}$ is assumed and the displacement response at the steady state can be represented by $\mathbf{X}_t = \mathbf{X}e^{i\omega t}$. Here, $\mathbf{P}$ and $\mathbf{X}$ denote the amplitudes of the excitation load and the response displacement, respectively. Thus, the equilibrium equation can be converted in the frequency domain as

$$(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})\mathbf{X} = \mathbf{P} \tag{5}$$

where $\mathbf{X} = \mathbf{X}_r + i\mathbf{X}_s$ is complex, and $\mathbf{X}_r$ and $\mathbf{X}_s$ donate the real and imaginary parts of the displacement amplitude, respectively.

3. Topology optimization formulation

3.1. Topology optimization problems

This paper will investigate the following four topology optimization problems for dynamic response of structures. Consider a dynamic structure under a harmonic load with a certain excitation frequency, the optimization objective is to suppress the vibration of the structure. The vibration of a dynamic structure can generally be measured by the amplitude of the response displacement. Following the dynamic compliance defined in Ma et al. [11], the dynamic compliance is the absolute product of the amplitudes of the excitation force and response displacement, $C_d = |\mathbf{P}^T\mathbf{X}|$. The first topology optimization problem for a dynamic structure under a given excitation load can be mathematically stated by

$$\text{P1}: \begin{cases} \min\ f(x_i) = |\mathbf{P}^T\mathbf{X}| = \sqrt{(\mathbf{P}^T\mathbf{X}_r)^2 + (\mathbf{P}^T\mathbf{X}_s)^2} \\ \text{s.t.} \begin{cases} V^* - \sum_{i=1}^{NE} V_i x_i = 0 \\ x_i = x_{\min} \text{ or } 1 \end{cases} \end{cases} \tag{6}$$

where V^* is the prescribed structural volume and V_i denotes the volume of the ith element. x_i is the design variable, which can take discrete values $x_{\min}$ or 1. $x_i = x_{\min}$ means that element i is void, where $x_{\min} = 10^{-3}$ is used throughout the paper to avoid the singularity. $x_i = 1$ means that element i is solid.

The above topology optimization problem highly depends on the excitation frequency of periodic loads. Previous numerical experience showed that the structure lost its integrity as the excitation frequency increased, and the resulting design failed to undertake any static load [26,27]. To eliminate this problem, one additional constraint on the static compliance can be included in the optimization problem. The topology optimization problem is therefore defined as