Engineering Structures 142 (2017) 192–210

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Truncation error analysis on modal flexibility-based deflections: application to mass regular and irregular structures

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ARTICLE INFO

Article history:
Received 21 June 2016
Revised 24 March 2017
Accepted 27 March 2017
Available online 7 April 2017

Keywords:
Flexibility truncation error
Modal flexibility-based deflection
Load participation factor
Uniform load
Mass proportional load
Mass irregular structure

ABSTRACT

It is of interest in the fields of vibration-based structural identification and damage detection to analyze the truncation effects introduced on modal flexibility (MF) based deflections that are estimated using only a subset of structural modes. To address this problem, an approach for truncation error analysis on MF-based deflections of structural systems subjected to a generic load is proposed in this paper. The approach is based on the determination of the relative contribution of each mode to the deflection by means of a proposed load participation factor (LPF). This factor, as derived analytically, depends both on the applied load and on the distribution of the structural masses. The validation of the proposed approach was carried out both on numerical models of shear-type frame buildings and on experimental data of a steel frame structure tested under ambient vibrations (i.e. the benchmark study sponsored by the IASC-ASCE Task Group on SHM). In both cases, results show that the LPF factors can give an a priori indication of the truncation effects expected on the MF-based deflections. The relationship between the proposed approach and the approach based on the mass participation factors, introduced by Zhang and Aktan (1998) for the case of uniform load (UL) deflections, is discussed since the two approaches are equal only if a special load, which is a mass proportional load (MPL), is considered. Thus, the application of this MPL load for mass irregular structures is also investigated. Numerical analyses performed both on a shear-type frame building and on a simply-supported beam, showed that for the great majority of the analyzed configurations, the truncation errors on the MF-based deflections due to the MPL are lower compared to those related to the UL.

1. Introduction

Recent advances in vibration-based testing technologies lead to the possibility of applying the techniques of the emerging field of Structural Health Monitoring (SHM) to existing civil structures and infrastructures. These testing techniques together with the application of structural system identification and damage detection algorithms can be adopted in order to evaluate possible structural deteriorations and support the decision-making on maintenance and retrofit programs.

Among different approaches for vibration-based damage detection (VBDD) [1–5], modal-based methods are founded on the principle that relevant damages in structures lead to changes in their dynamic properties. Such properties (i.e. modal parameters) can be derived using modal identification techniques from the measurements of ambient or forced vibrations of the structures [6–8]. Moreover, modal flexibility (MF) matrices can be assembled starting from identified modal data and, as shown in [9], the coefficients of such matrices are more sensitive to damages than the modal parameters individually. Due to this, promising VBDD methods are based on the calculation of MF matrices [10–13], including methods specifically developed for the case of output only identification [14–16].

As defined by Zhang and Aktan [17], the uniform load (UL) surface is the deflection profile that can be calculated from experimental MF matrices by applying uniform loads at all the DOFs of a structure. This surface can be adopted as a damage-sensitive feature [17–19] since eventual modifications in the UL deflection, when comparing at least two structural

Abbreviations: LPF, load participation factor; MF, modal flexibility; MPF, mass participation factor; MPL, mass proportional load; MPL-PF, mass proportional load participation factor; STFB, shear-type frame building; SSB, simply-supported beam; UL, uniform load; UL-PF, uniform load participation factor.

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http://dx.doi.org/10.1016/j.engstruct.2017.03.057
0141-0296/

states, can be related to structural damages, such as stiffness reductions. Similar approaches have been specifically developed for the damage detection of shear-type buildings [20–23] using special loads, termed positive shear inspection loads. These methodologies have also been applied to beam-like structures [24].

In modal identification of civil structures the high-order modes cannot be extracted or are poorly estimated. Thus, the use of such incomplete modal models [25] for assembling modal flexibility (MF) matrices leads to inevitable errors, which are defined as flexibility truncation errors [17], with respect to static flexibility matrices. These errors affect also the uniform load deflection which is calculated from MF matrices. As also defined in [17], truncation error analysis is related to the investigation of how many modes need to be included in order to obtain good estimates of the MF matrix and the UL deflection derived from experimental data. This analysis is commonly adopted in pretest design using analytical or numerical models in order to determine the frequency range to be tested, so that the truncation errors can be minimized [17].

As indicated by Zhang and Aktan in [17], there exist three main approaches for truncation error study: the first one is to consider a number of modes such that the cumulative mass participation factor of the structure is higher than a selected threshold; the second approach is to directly compare the truncated MF matrix with the exact matrix; the third one is equal to the second approach but the UL deflection is considered instead of the MF matrix. The mentioned authors [17] performed a series of analyses using the second and the third approaches, on a 10 DOF mass-spring model and a bridge FEM model, demonstrating that the UL deflection is less sensitive to truncation errors than the MF matrix.

The first objective of the present work is to propose an approach for truncation error analysis that is applicable to modal flexibility-based deflections of structural systems subjected to a generic load. The main advantage of the proposed approach is that it does not imply a direct comparison between the truncated and the exact solutions, and it can be used to obtain an a priori estimate of the truncation effects expected on the deflections. In particular, it is demonstrated that the proposed approach is equal to the first approach for truncation error analysis, not applied in the numerical analyses of Zhang and Aktan [17], only if a special load, which is a mass proportional load (MPL), is considered. The second objective is then to compare the proposed mass proportional load with the uniform load [17], by evaluating the corresponding truncation errors on the MF-based deflections of structures characterized by mass irregularities.

The paper is organized as follow. At first the proposed general approach for truncation error analysis is outlined, and then the analytical expressions for evaluating the truncation errors on the interstory drifts of the MF-based deflections of shear-type frame buildings (a special case) are derived. Secondly, the validation of the proposed approach, carried out both using numerical simulations and experimental data, is presented. Numerical models of shear-type frame buildings were employed for this validation. In addition, the approach was verified on a steel frame structure tested under ambient vibrations, using the experimental data of the benchmark problem (phase II) provided by the IASC–ASCE Task Group on Structural Health Monitoring [26,27]. Finally, numerical analyses performed to compare the truncation errors on the deflections due to the MPL load with those related to the UL load are presented. These analyses were conducted on mass irregular structures, considering both a shear-type frame building and a simply-supported beam.

2. Theoretical background

2.1. Modal flexibility matrix and uniform load deflection estimated from vibration data

According to the equations of motion and the dynamic characteristic equations of an undamped MDOF structural system, the modal flexibility matrix $\boldsymbol{F_r}_{\,n\times n}$ can be expressed using r modes as

$$\boldsymbol{F_r} = \boldsymbol{\Phi_r}\boldsymbol{\Lambda_r}^{-1}\boldsymbol{\Phi_r}^T \tag{1}$$

where $\boldsymbol{\Phi_r}_{\,n\times r}$ is the mass-orthogonal and mass-normalized mode shape matrix, $\boldsymbol{\Lambda_r}_{\,r\times r}$ is the spectral matrix, which contains the square of the first r natural circular frequencies ω_i^2 on the main diagonal, $i = 1\ldots r$ is the mode index, and n is the number of the DOFs. Each component of the matrix is expressed as

$$f_{j,k} = \sum_{i=1}^{r} \frac{\phi_{j,i}\phi_{k,i}}{\omega_i^2} \quad \text{with } j,k = 1\ldots n \tag{2}$$

Each term in the summation is independent from the global sign of the i-th mode shape, i.e. it does not change if the mode shape is multiplied by -1, and only the diagonal components of the matrix are always positive quantities [17]. If $r = n$ and considering exact modal data, the static flexibility matrix $\boldsymbol{F_n}$ is obtained. If $r < n$ each component of the matrix is approximated with respect to the exact value [25], and the difference between the two terms is called residual flexibility [28]. Nevertheless, accurate estimations of the flexibility matrix can be obtained using the first modes since the contribution of each i-th mode to the matrix depends on the term $1/\omega_i^2$ [15,25]. Adopting Eq. (1), experimental modal flexibility matrices can be assembled starting from modal parameters extracted using system identification techniques [6–8] from the measurements of ambient or forced vibrations of structures. Several procedures can be applied in order to obtain the required mass-normalized mode shapes both for the case of input-output [7,29] or output-only modal identification [14,15,20–23,30,31]. For the particular case of structures with closely spaced modes, a multiple input test and suitable modal identification techniques, available in the literature, are required to extract the structural modes [8]. However, as described in [32,33], the case of closely spaced modes inherits some relevant properties from the theoretical case of repeated eigenvalues. One of these properties is that, according to the dynamic characteristic equations of an undamped system, the orthogonality relation between two modes characterized by the same eigenfrequencies is not assured [32,33]. A similar property may thus be related to identified closely spaced modes, and issues may arise if modes characterized by such property are employed to calculate the modal flexibility matrices using Eq. (1). With reference to this last point, however, it is worth noting that there exist examples in the literature where the modal flexibilities were adequately estimated in presence of closely spaced modes (one of these examples is reported in [34]).

The deflection of an MDOF structure calculated starting from the modal flexibility matrix $\boldsymbol{F_r}$ and due to a uniform load (UL) $\boldsymbol{u} = [1\ 1\ \ldots\ 1]^T$ is defined as the uniform load surface [17], and it is expressed as $\boldsymbol{x_{u,r}} = \boldsymbol{F_r}\boldsymbol{u}$ where the j-th component of the deflection vector $\boldsymbol{x_{u,r}}$ is

$$x_{u,r,j} = \sum_{i=1}^{r} \frac{\phi_{j,i}}{\omega_i^2}\left(\sum_{k=1}^{n}\phi_{k,i}\right) = \sum_{i=1}^{r} \frac{\phi_{j,i}}{\omega_i^2}s_i \tag{3}$$

The convergence of the UL deflection to the exact solution is more rapid with respect to the components of the modal flexibility matrix due to the presence in Eq. (3) of the term $s_i = \sum_{k=1}^{n}\phi_{k,i}$. Indeed, the summation of the i-th mode shape coefficients can be considered as the contribution of that mode, and in general higher