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Discrete element analysis of gothic masonry vaults for self-weight and horizontal support displacement

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ABSTRACT

The paper investigates how the deviation of the centreline from the semi-circle influences the mechanical behaviour of gothic masonry vaults. The geometry of the vaults is quantified by a scalar parameter. The present work deals with the force-transmitting system, crack opening magnitude and pattern. Furthermore, the magnitude of the so-called threshold support displacement and the magnitude of the support displacement at first failure is also analysed. Masonry structures consist of separate blocks, each of which can move independently from each other, which makes the calculation process rather complicated. The Discrete Element Method (henceforth DEM) is used to solve the highly nonlinear problem. In the chosen method the equations of motion are solved by the central difference scheme. Approximate boundary conditions are applied, which assume symmetry in the analysed structures. The main findings of the work are: barrel vaults with pointed shape become statically determinate at a higher support displacement level than barrel vaults with semi-circular centreline. In the case of large deviation from the semi-circle or at large support displacement level shear failures can appear in gothic masonry barrel and cross vaults, which could decrease the magnitude of feasible horizontal support displacement.

1. Introduction

Masonry arches with pointed shapes were really widespread in the gothic architecture. Pointed arches have preliminaries in the Islamic architecture and the idea may have been brought to the Christian Europe from Moorish Spain or from the Near East by the crusaders. The beauty of the pointed arches unquestionably helped the spreading of this type of structural shape in the cathedral art of the Christian world from the 12th century. But it is evident that such a fast spreading of gothic masonry vaults could not evolve without clear mechanical advantages.

The history of the analysis of masonry vaults from the Trial-and-Error technique through the Rules of Thumb to the Limit State Analysis are summarised in the work of Gaetani et al. [1]. Nowadays there are many methods, which help to analyse structures with no-tension materials, for instance see Limit State Analysis (henceforth LSA), Finite Element Method (henceforth FEM) and DEM. The description of LSA can be found in Huerta [2] and in Heyman [3]. There are many valuable works, based on this approach, see the work of Kooharian [4] and the work of Cocchetti et al. [5], which are applications in 2D. LSA in 3D can be found in the study of Block et al. [6] and in the paper of Fraternali [7]. In the research of masonry structures DEM is a new, spreading approach, see the extensive work of Sarhosis et al. [8]. For 2D and 3D applications of DEM see the following works: [9], where the collapse modes, minimum thicknesses of circular masonry arches are analysed analytically and numerically. In [10] the collapse modes were analysed of a groin vault with DEM and with physical tests. In [11] the role of the ribs in masonry groin vaults was analysed by DEM.

In the LSA of the load bearing capacity of masonry structures the main unknown is an objective function, which is a load multiplier with constraints on the internal forces, that yields a linear programming problem, see for instance [12,13] or [14]. While in a deformable DEM model the basic unknowns are the nodes' translational degrees of freedom. In LSA only the state of collapse is under examination, while in a deformable DEM model the displacements, internal force transmitting system are followed during the loading process.

It was shown with LSA that pointed arches express significantly smaller lateral thrust to their supports than semi-circular vaults, see the work of Ochsendorf and Romano [15], Lengyel and Bagi [16]. In the last work the horizontal reaction component of barrel vaults was analysed analytically and numerically with DEM for self-weight and for different magnitudes of horizontal support displacement. In the paper of Rosa and Galazia [13] it was shown with LSA that the equilateral pointed arch is the most resistant

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for self-weight and for live loads. The minimum thickness of pointed arches and the lateral load-bearing capacity were carried out by LSA in Cavalagli et al. [14]. The author analysed the minimum thickness of the pointed arch with LSA and with DEM in [17]. The same problem was analysed with graphical simulations and with LSA in the papers of Nikolic, see [18,19]. The equilibrium and collapse modes of pointed arches were analysed in the papers of Aita et al. [20–22], where the Durand-Claye's method is applied to solve the problem with piecewise linear constitutive relation. Further applications of this method can be found in Foce and Sinopoli [23] and in Foce and Aita [24]. Analysis of gothic cross vaults with FEM was carried out in Valcárcel et al. [25]. Investigation of complex gothic structures was done with 3D finite element damage model by Roca et al. [26]. A case study on a romanesque type of cross vault was made with nonlinear FEM in Milani et al. [27]. Discrete element analysis with rigid blocks was carried out by McInerney and DeJong [28]. The authors analysed the displacement capacity of barrel and cross vaults for horizontal and vertical support displacements. In this work the supporting effect of the adjacent structural elements on groin vaults was neglected and the ribs were not modelled. Furthermore, due to the application of rigid elements the Poisson effect was not taken into account, which may have a significant effect on the resistance against sliding failures.

The subject of the research is the analysis of gothic barrel and cross vaults, see Fig. 1. The question of interest is what are the advantages of the application of the pointed shape. For instance, can the pointed shape increase the magnitude of feasible horizontal support displacement of groin vaults or the opposite, it weakens the structure. The methodology of the problem is: the shapes of the vaults are quantified by a scalar parameter and the highly nonlinear problem is solved by DEM, where cracking, separation and sliding failure can freely develop. In the applied DEM code large numerical damping is used, which decreases the accelerating forces by 80% in every time step in order to obtain the equilibrium solution. The equations of motion are solved by a numerical time integration technique, with the so-called central difference scheme. This numerical approach is not unconditionally stable, so efficiently small time step is applied in order to obtain convergence.

The study can be classified into two parts:

- Analysis of the internal force-transmitting system of gothic barrel vaults for self-weight. As a structure's geometry is never perfect in reality, investigations are carried out for horizontal support displacement with different magnitudes too.
- An appropriate, but approximate boundary condition system for cross vaults is set up in order to study the force-transmitting system in the masonry shell and in the diagonal rib. Furthermore, the crack opening magnitude, pattern and the magnitude of the maximal feasible horizontal support displacement are also the subjects of the research.

To the knowledge of the author discrete element analysis with elastic blocks on the force transmitting system of gothic vaults considering the shape of the structure was not carried out yet. Furthermore, 3D analysis of ribbed gothic cross vaults with non-continuous, discrete model, where the supporting effect of the adjacent structural members is taken into account was not found in the literature by the author.

In a previous work of the author with a co-author in [16] the analysis of the reaction components of gothic barrel vaults for self-weight and horizontal support displacement was carried out, but the study did not consider cross vaults, internal force-transmitting system and crack pattern. Evidently, the results, obtained here with the restriction on the boundary conditions can be expanded for different spans, if the ratios of the geometrical quantities to the span are the same.

The paper is organised as follows: Section 2 introduces the fundamentals of the applied DEM code, furthermore deals with the main assumptions, notations, geometrical, material parameters, boundary conditions and introduces the analysed characteristics. Section 3 contains the numerical results. Finally, in Section 4 the main conclusions are drawn.

2. Description of the analysed problem

2.1. Fundamentals of 3DEC

The discrete element analysis is done with software 3DEC, developed by Itasca CG. In this code the discrete elements may have convex or concave polyhedral shape. On the one hand the elements can be rigid in this case they do not have any deformation or on the other hand if the elements are deformable, then they behave according to Hooke's law. The elastic deformations of the blocks are taken into account in the considered models. This approach of the problem is important for the sophisticated analysis of the shear failure. In a masonry shell the Poisson effect can be significant, because it has a large effect on the distribution of the pressure forces, which may have a significant influence on the resistance against shear failure.

The user gets deformable model by dividing the discrete elements into uniform strain tetrahedral finite elements. The material parameters of these elements must be specified by the user. The nodes of a tetrahedron have only translational degrees of freedom. The basic unknowns of the analysis are the nodal translations (three unknowns for each and every node). In a deformable model

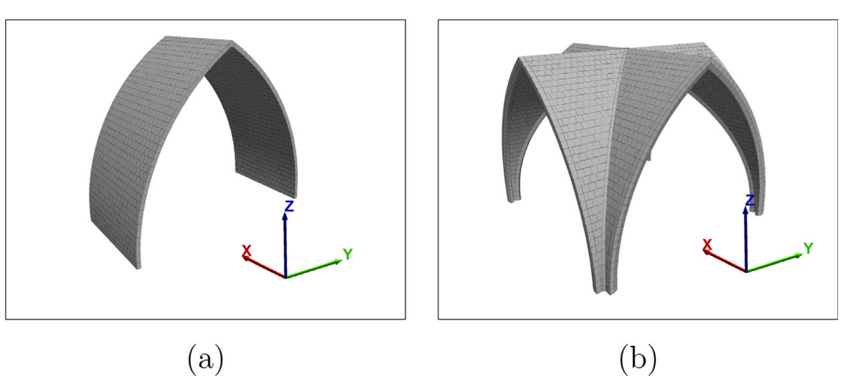

Fig. 1. The analysed models: barrel vault (a), cross vault (b).