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Flexural strength-ductility assessment of unreinforced masonry cross-sections: analytical expressions

Nicola Giordano *, Pietro Crespi, Alberto Franchi

Department of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy

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A B S T R A C T

The unreinforced masonry (URM) is a complex and variegate construction material characterized by a prominent nonlinear response. For this reason, advanced numerical simulations are required to assess URM buildings, especially in case of severe loading conditions as earthquakes.

However, given the theoretical and computational difficulties of detailed non-linear analyses, linear elastic methods are still adopted in current practice. This results in conservative seismic assessments and, consequently, invasive and expensive strengthening interventions to guarantee seismic safety.

Starting from these statements, the aim of the paper is to provide closed-form equations useful for a preliminary strength and ductility assessment of unreinforced masonry rectangular cross-sections. Expressions for direct calculation of M-N (bending moment – axial load) strength domains and M-χ (moment – curvature) ductility diagrams for different constitutive laws are provided. The expressions are firstly applied to a representative URM cross-section and secondarily used for the numerical simulation of a recent out-of-plane loading experimental test available in the literature. For a better comprehension of URM members behavior under axial-bending loading condition, 3D M-N-χ diagrams are presented in the paper.

1. Introduction

Recent seismic events (L'Aquila Earthquake, Italy 2009 [1], Emilia Earthquake, Italy 2012 [2], Napa Earthquake, California 2014 [3]) have demonstrated another time that existing masonry buildings are affected by significant structural deficiencies. Poor quality of the materials, geometrical irregularities, inadequate wall-to-wall connections and absence of anti-seismic detailing are just a few aspects of masonry constructions weaknesses. Over the last years, the technical-scientific community has paid serious attention to the problem focusing on two main research lines. (i) Experimental investigation of the structural response of masonry before and after the application of strengthening interventions, both as prior-strengthening and as repair methods after damage. (ii) Development of analytical and numerical tools suitable for the seismic assessment and retrofitting design process of URM structures.

On one hand, the large amount of experimental tests carried out on masonry constructions [4] has allowed qualitative and quantitative validations of strengthening techniques (i.e. mortar injections, carbon-fiber reinforcements, tie-rods installation, floor-diaphragm strengthening, etc.) and calibration of masonry stress-strain constitutive models. On the other hand, the reliability of seismic assessment methods for URM buildings is still matter of debate [5], as recently observed in a blind test predictions experimental project carried out by Mendes et al. [6]. As a matter of fact, the usual hypotheses adopted in the structural analysis of reinforced concrete (r.c.) and steel buildings are no longer valid for masonry, as briefly reported in Table 1.

Lourenço et al. [7,8] furthermore underlined how the problem of knowledge is central when we are dealing with the seismic assessment of existing masonry constructions. In most cases the building geometry implemented in the structural software is not supported by a precise survey. Also, the internal composition of the walls cannot be investigated in an exhaustive way because of historic conservation prescriptions. Moreover, the characterization of the mechanical properties of the materials is challenging and requires expensive in-situ tests (flat jacks, diagonal compression, non-destructive tests, etc. [9,10]). Finally, especially for historic buildings, the material state of stress is influenced by the variability of the mechanical properties, by the construction stages and by the continuous human modifications and repairing interventions occurred during the life of the structure [11].

* Corresponding author.
E-mail addresses: nicola.giordano@polimi.it (N. Giordano), pietro.crespi@polimi.it (P. Crespi), alberto.franchi@polimi.it (A. Franchi).

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Table 1
Usual hypotheses adopted for the assessment of general structures and differences from masonry structures.

	General Structures	Masonry Structures
Material/structural components behavior in Service Limit State (SLS)	Linear elastic.	Linear elastic response in compression.Very low resistance in tension (no-tension material assumption).
Material/structural components behavior in Ultimate Limit State (ULS)	In general, it is possible to adopt elastic-plastic constitutive models in tension/compression. Structural component damages are usually concentrated in plastic hinge regions.	Material behavior in compression is characterized by a softening branch. Structural components can lead to collapse for bending damage, shear damage or loss of equilibrium.
Modelling	The structure (usually a 3D frame) is represented by a beam finite element model.	The structure is composed by a masonry continuum which, in some cases, cannot be discretized as a system of beam elements.
Type of analysis	Response Spectrum Analyses (RSA) are recommended by codes and guidelines.	Since elastic analyses cannot estimate the redistribution of stresses due to cracking, nonlinear methods are required.
Behavior under seismic actions	Global behavior is guaranteed by good node connections between structural elements.	In case of poor wall-to-wall/wall-to-floor connections, extensive cracks and damages can lead to the collapse of entire portions of the building (collapse mechanism).

Depending on typology of the building, availability of mechanical and geometrical data and expected computational-cost, different seismic assessment approaches have been proposed in the literature:

– Macro-Element Method or Equivalent Frame Method. The building is subdivided into beam elements (piers and spandrels) connected in the intersections with rigid nodes. The deformable elements are characterized by in-plane behavior and the non-linear response is concentrated in appropriate plastic hinges that take into account the geometry of the masonry member and the mechanical properties of the material in shear and compression (Lagomarsino et al. [12]).
– *Continuum Finite Element Method (FEM)*. When the structure is characterized by a complex geometry, it is hard to adopt an equivalent frame simplification. In these cases, advanced FEM models with 2D/3D elements are preferred. Usually three approaches are adopted. I) Detailed micro-modelling: bricks and mortar joints are represented by continuum elements. II) Simplified micro-modelling: bricks are modeled by continuum elements while the behavior of the mortar joints is lumped in discontinuous interface elements. III) Macro-modelling: bricks and mortar are smeared out in a homogeneous continuum [7].
– *Discrete Element Method (DEM)*. The masonry structure is subdivided in a discrete number of rigid bodies. The methodology is formulated in large displacement and enables finite displacements and rotations of these bodies including complete separation; new contacts are also caught by the method (Lemos [13]).

The above mentioned techniques have been continuously developed and compared by academic researchers [14,15]. Despite that, consulting engineers and practitioners are loath to adopt complex non-linear methods. High computational costs, complex theoretical aspects, questionable interpretation and validation of the results, are just a few of the problems underlined in current practice. Unfortunately, the effect of these perplexities results in an improper use of linear elastic analyses that cannot catch the unavoidable cracking response of the masonry under seismic loads [16].

1.1. Assessment of URM cross-sections: Literature review

Generally speaking, damages in URM components are a combination of three physical phenomena [17,18]: cracking of the resisting cross-section and toe compression failure generated by bending moment; bed joint sliding produced by shear forces; diagonal cracks due to shear actions. However, in case of slender masonry elements (such as columns, walls in out-of-plane loading, slender piers in in-plane loading, etc.), the leading failure mode is the one governed by bending. It is important to underline that the crisis at the sectional level (toe compression failure) can be preceded by the loss of equilibrium of the masonry element. This phenomenon occurs when the compressive strength of the masonry assemblage is relatively high or when the vertical stress due to dead loads is low.

Given this range of applicability, since '70ies [19] researchers have developed beam-theory based calculation tools able to predict the seismic capacity of URM elements. These studies, on one hand gave the starting point for the equivalent frame method formulation [20], on the other hand evolved in a specific research line focused on the evaluation of the in-plane and out-of-plane capacity of URM members via non-linear cross-section analysis [21,22].

The principal assumption of the cross-section analysis is that axial strains behave linearly in bending i.e. sections remain plane. The advantages and the limitations of this hypothesis has been deeply discussed and validated [16,23,24]. Looking at the experimental data collected by Brencich et al. [25] and Cavaleri et al. [26], the plane section assumption better fits in case of slender walls, regular masonry blocks and absence of rubble masonry internal leaf.

Results of the cross-section analysis are usually summarized in two diagrams: the *M-N* (bending moment - axial load) interaction curve reports the strength limit of the section; the M-χ (bending moment - flexural curvature) curve describes the deformation capacity of the section for a given axial load or eccentricity. Due to the complexity of the masonry material behavior, the analytical derivation of the *M-N* and M-χ diagrams is matter of interest for the scientific community.

In 2013 Parisi et al. [23] evaluated *M-N* strength domains for different masonry constitutive models. In their study $M = f(N)$ closed-form equations for parabola-rectangle EC6 law [27] are firstly derived. Then, the effect of strain-softening is investigated adopting two advanced masonry models by Turnšek-Čačovič [28] and by Augenti-Parisi [29]. Depending on the complexity of the analytical problem, the resulting *M-N* curves are described by closed-form expressions or by a set of non-linear equations solved numerically.

The analytical estimation of M-χ curves has been investigated mainly for the case of constant eccentricity of the axial load. La Mendola [30] derived moment-curvature diagrams adopting the stress-strain softening model by Naraine et al. [31]. In their work, the nonlinear equations that define the equilibrium and compatibility of the cross-section are solved using an iterative numerical procedure. A similar study was carried out by Cavaleri et al. [26] adopting the Sargin concrete model [32]. More recently Parisi et al. [24] investigated the impact of different stress-strain models on M-χ curves. The results are compared to experimental M-χ