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Large displacement analysis of dry-jointed masonry structures subjected to settlements using rigid block modelling

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ABSTRACT

A discrete element model is proposed for large displacement static analysis of dry-jointed masonry structures subjected to foundation settlements. The numerical model is a two-dimensional assemblage of rigid blocks interacting at no-tension, frictional contact interfaces with infinite compressive strength. The static contact problem associated to the proposed model is formulated as a linear mathematical programming problem and an incremental procedure is implemented to take into account large displacements. Experimental tests were carried out on small scale panels with opening subjected to imposed settlements to validate the proposed model. Comparisons in terms of failure modes and maximum allowable support displacements just before the collapse are presented. An experimental case study from the literature related to a masonry arch on spreading support is also analyzed for validation. On the basis of the obtained results, the accuracy and the computational efficiency of the formulation adopted are discussed.

1. Introduction

This paper is devoted to the development of a discrete rigid block model for the assessment of dry-jointed masonry structures subjected to foundation settlements or spreading support in the large displacement regime. The proposed model takes into account both opening and sliding failure at block interfaces. The numerical formulation was developed in the framework of incremental limit equilibrium analysis and linear programming was used to solve the associated contact problem. As such, few geometrical and mechanical parameters govern the response of the numerical model, namely rigid block configuration, load distribution and friction coefficient, leading to efficient and robust computational solutions.

Crack patterns and distortions induced by foundation settlement or spreading supports represent one of the most frequent causes of damage in masonry structures.

Evaluating consequences associated to foundation or support movements, both in terms of damage (i.e. cracks width) and collapse (i.e. amount of support displacements involving loss of stability), is one of the main questions that, in the past decades, has interested architects and engineers dealing with the assessment of historical and existing masonry constructions [1].

Emblematic examples that posed significant challenges for builders are the differential foundation settlements of Venetian buildings caused by soft soils [2], the settlement mechanisms in the naves of the Cathedral of Milan due to subsidence [3] and in the Cathedral of Agrigento due to slope instability problems [4], just to cite a few [5].

Other notable examples of failure mechanisms caused by support movement are frequently concerned with vaults and domes on spreading supports [6,7], which in these cases can be also related to inherent structural deficiencies in addition to soil deformation problems.

From the computational point of view, the prediction of the effects induced by imposed movements on masonry structures can be made by using a variety of modelling approaches.

Those include finite element [8–16] and distinct element models [17–20] as well as rigid body spring models [21–23]. As an alternative to the above-mentioned modelling methods, limit analysis (LA) can be also conveniently used to investigate the behavior of masonry structures subjected to settlements. When compared to other computational modelling approaches, the use of LA formulations is mostly attractive for its ability to assess failure modes and associated loads directly, with no iterative calculation steps and simple assumptions on mechanical properties of constituent materials [24].

Several analytical and computational models based on the upper and lower bound formulations of LA theorems have been proposed in the literature. Analytical formulations based on thrust line analysis methods as well as numerical software tools for the analysis of masonry arches subjected to spreading supports can

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be found in [25–29]. Strut-and-ties models and load path method have been proposed in [30–32] for the interpretation of the crack patterns in the case of landslides-induced settlements.

Discrete element models based on fundamental theorems of limit analysis, such as the rigid block models developed in [33–41], can be also used to evaluate the crack patterns and to investigate the likely causes of damage observed on dry-jointed masonry structures subjected to settlements [42–44].

It is well known that, in the case of limit analysis formulations, the solutions obtained are expressed in terms of infinitesimal displacements. As such, when the objective of the analysis is to predict the amount of support movement promoting the collapse, a key issue is concerned with numerical techniques adopted to take into account large displacements. Generally, these numerical procedures are used to magnify infinitesimal displacements which are obtained from limit analysis formulations in order to determine new position and contact configuration of the blocks.

In the framework of limit analysis formulations, different incremental solution procedures have been proposed in the literature to take into account the effects of large displacements [45].

Most of these procedures have been developed for numerical models derived from analytical formulations with reference to special structural configurations (i.e. thrust line analysis method applied to circular arches) and under simplified assumptions for failure mechanism (i.e. hinging or rocking with sliding prevented). Examples of incremental solution procedures based on static and kinematic approaches for the large displacement analysis of masonry arches subjected to spreading supports can be found in [25,26,29]. In this context, it is worth noting that incremental solution procedures have been also proposed in [46] for non-linear static analysis of masonry buildings using strut and tie models. In this case evolutive models are used for in-plane loaded wall panels to take into account interlocking and friction dissipation as well as large displacement effects.

To overcome limitations related to structural configuration and failure mechanisms, rigid block models adopting incremental solution procedures could be also used, considering that in this case sliding and crushing can be included in the formulation of failure conditions in addition to hinging/rocking failure.

Large movements effects using a computational rigid block model were addressed by Gilbert in [47] to model soil-structure interactions in the case of masonry arch bridges and tunnel linings. The Author proposed a coupled analysis procedure which involves updating the geometry and soil pressures at successive iterations.

In the framework of rigid block models, an attractive and promising method to deal with configuration changes in gross displacement analysis makes use of gap functions for the formulation of non-penetration conditions at contact points. The use of such functions is well consolidated in non-smooth contact analysis [48–51], and recent applications to the case of masonry structures under seismic loads can be also found in [52]. However, to the best of the authors' knowledge, no application exists of these formulations to the case of large displacement static analysis of masonry structures subjected to support movement.

With that in mind, the present study is aimed at developing a simple rigid block model for large displacement analysis of dry-jointed masonry structures. The proposed model relies on an incremental limit equilibrium analysis formulation using gap functions for the prediction of failure mechanisms and for determining the amount of support movement promoting the collapse.

A key feature of the proposed formulation is related to the simplified procedures which have been adopted for the specification of potential contact points and for the updating of the gap functions to take into account gross-displacements and to analyze the response of large numbers of rigid blocks with contact and friction. As such, the formulation adopted represents an extension to the case of large displacements of the computational model presented in [44], where infinitesimal displacements have been assumed to investigate the behavior of masonry block structures subjected to settlements.

The objective of this study was to assess the ability of the proposed formulation in predicting the behavior of masonry structures subjected to foundation settlement by comparing numerical and experimental outcomes, and to evaluate the accuracy and computational efficiency of the adopted modelling approach when applied to large scale wall panels.

The paper is organized as follows. The relationships governing the behavior of the rigid block model and the contacts formulation are presented in Sections 2 and 3. The adopted iterative solution procedure for large displacements is described in Section 4. In Section 5 a validation study is presented against ad-hoc experimental investigation involving small-scale masonry portal frames. Experimental results from the literature on a circular masonry arch subjected to spreading support are also used for validation. To assess the efficiency of the proposed model when applied to assemblages with a large number of blocks and contacts, in Section 6 the formulation is applied to numerical case studies of 2D wall panels with different load configurations.

2. The rigid block model for large displacement analysis

To represent the masonry structure subjected to settlements or spreading supports, we assume an assemblage of rigid blocks i interacting at no-tension, frictional contact interfaces j with infinite compressive strength (Fig. 1).

The rigid block behavior is governed by equilibrium equations, relating external and internal (contact) forces, and by kinematic equations, which ensure compatibility between contact displacement rates and block degrees of freedom.

To model the interface behavior, failure criteria are defined on the basis of contact forces and flow rules are adopted to express the increment of displacement rates when failure occurs.

In order to apply base movements to the masonry panels, an additional support block s is introduced in the numerical model (Fig. 1a). Although, in principle, different support displacement profiles could be modelled using the present formulation, in this study we assume that the settlement is governed by a single degree of freedom which is associated to this support block (i.e. vertical, horizontal or rotational movement).

On the basis of the relations governing the rigid block assemblage, upper and lower bound problems are formulated and solved for the calculation of the failure mechanism and contact forces associated to the imposed movement.

Large displacements are taken into account implementing a simple sequential solution procedure which is divided into increments to determine new positions of rigid blocks and contacts configuration, as detailed in the following sections.

2.1. Contact forces and kinematic variables

A point contact model is adopted to represent interactions at interfaces j. The primary static variables are the internal forces acting at each contact point k, which are located at a vertex of interfaces j (Fig. 2a). These variables are collected in vector $\boldsymbol{c_k}$ and include the shear force component t_k and the normal force n_k along the local coordinate axes:

$$\mathbf{c}_{\boldsymbol{k}} = [\, t_k \quad n_k \,]^{\mathrm{T}}. \tag{1}$$

Following usual limit analysis formulations, external loads $\boldsymbol{f}$ are expressed as the sum of known dead loads $\boldsymbol{f_D}$ and live loads $\boldsymbol{f_L}$ which act at the centroid of the blocks.