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Bayesian approach to breathing crack detection in beam structures

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ABSTRACT

In this paper, a Bayesian approach is developed to conduct uncertainty quantification on a single breathing crack in a beam structure using nonlinear forced responses. The proposed methodology not only determines the breathing crack characteristics but also quantifies associated uncertainties of the inferred values. Such information is important for fatigue crack monitoring and remaining life prediction in cracked beam structures. First, a single degree of freedom model is developed to characterize the nonlinear behavior of the cracked beam. The Modified Homotopy Perturbation Method (MHPM) is applied to determine analytical approximate solutions. Then, a Bayesian inference approach is proposed by applying Markov chain Monte Carlo (MCMC) technique, in which the Random Walk Metropolis algorithm is employed. The objective is to estimate crack size or location from the nonlinear vibration responses, in which noise is added to represent actual measurement data. Finally, the proposed probabilistic damage detection approach is successfully demonstrated and the breathing crack status is quantified with associated uncertainties. This leads to a new way of detecting a single breathing crack in beam structures.

1. Introduction

The presence of a transverse edge crack in a structural element introduces a local flexibility which has a potential to reduce the load carrying capability of the element. Also, many structural elements in engineering applications are designed to operate under cycling stresses. A crack will experience opening and closing in the time following the external loading excitation characteristics. Early detection of such crack will prevent further crack propagation and potential failure in the structure. Vibration-based detection is a powerful method to conduct this non-destructive evaluation for the cracked structures. The first step is to understand the crack's effect on a structure element and subsequently determine the response of such cracked structure.

Irwin related the flexibility to the stress concentration factor for a cracked beam [1]. The idea of using the stress concentration factor as an overall factor became a standard method to relate the local compliance to the strain energy. Both analytical and experimental results were tabulated under a wide range of loadings, crack conditions, and geometries [2]. Dimarogonas and Paipetis derived the local stiffness matrix or flexibility matrix of a beam with a transverse crack under general loading conditions based on Castigliano's theorem and law of fracture mechanics [3]. A special case of the symmetric double-sided crack in a Bernoulli-Euler beam was examined by Christides and Barr [4]. The variational principle was applied to formulate the governing equation and the crack effect on the stress was assumed to decay exponentially from the crack location. Experiments were conducted to validate the first natural frequency of the cracked beam. Also, a finite element model of a cracked prismatic beam was developed [5]. It was found that the presence of moderate size cracks has significant effects on the natural frequencies and vibration amplitudes of the beam. The local flexibility method has been applied to study different cracked structures [6–9]. However, among all the above research, the crack is assumed to be open and remains open even under a cyclic loading. Such an assumption is made in order to avoid dealing with nonlinear equations of the cracked structures.

To account for the opening and closing of a crack, we must investigate the fatigue crack model and determine associated nonlinear vibration response. The fatigue crack is also called a breathing crack. Basically, it is open when the crack is under tension. Otherwise, it is closed. Typically, a structure with a breathing crack is modeled as a bilinear oscillator, i.e. two distinct stiffness constants to account for the fully open and the closed state. Shen and Chu [10] derived the bilinear (nonlinear) equation of motion and associated boundary conditions for a uniform beam with a single fatigue crack. The dynamic responses were determined numerically for a simply supported beam. In a subsequent paper by the

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Nomenclature

α	stiffness ratio	ν	Poisson's ratio
$\alpha(q^*\|q_{k-1})$	probability of accepting candidate q^*	Ω	forcing frequency [rad]
c	crack depth [m]	π_0	prior probability density function
c_{ii}	flexibility influence coefficient [m/N]	π	posterior probability density function
chol	Cholesky factorization	q	random variable for crack depth or location
E	Young's modulus [GPa]	R	*chol* (V)
F	amplitude of forcing function [N]	σ^2	error variance
f	normalized amplitude of forcing function	SS_q	sum of squares error
γ	first mode shape function	$S(u)$	restoring force function
gamrnd	gamma random numbers	$u(t)$	tip displacement
h	height (thickness) of the beam [m]	u_α	state variable
K_c	stiffness of uncracked beam [N/m]	V	covariance matrix
K_o	stiffness of cracked beam [N/m]	v_{obs}	measurement data
K_{15}	cracked beam stress intensity factor [Pa m$^{1/2}$]	w	width of the beam [m]
L	length of the beam [m]	ω_0	linear undamped natural frequency
L_c	distance from tip of beam to crack [m]	ω	forcing frequency
MI	bending moment	ω_L	linear frequency of cracked beam (crack is closed) [rad]
m	lumped mass of the beam [kg]	Z	space of parameter test functions
$\bar{m}$	mass per unit length [kg/m]	ζ	damping ratio
p_i	tip force		

same authors [11], a closed-form solution for such bilinear oscillator was developed using two square waves to represent stiffness changes under low-frequency excitation. Nonlinear behaviors were observed in the time history and frequency spectrum under a quite large crack, in which the crack depth is 50% through the beam thickness. Sundermeyer and Weaver [12] studied the dynamic response of a bilinear spring-mass system that was used to model a beam with a breathing crack. The purpose of this study was to determine the crack location, depth, and opening load based on the crack signatures from the Fourier transformation of time history response excited at two frequencies. Chati et al. [13] focused on the modal analysis of a cracked beam. By modeling the cracked beam as two linear configurations (crack open and closed), they defined the effective natural frequency (also called breathing frequency). The finite element method was used to compute the bilinear frequency of a cracked beam by solving each piecewise-linear system. Also, a two degree of freedom model was used to understand cracked beam dynamics using the perturbation method. Rivola and White [14] applied a simple single degree of freedom model to a beam with a closing crack and conducted bispectral analysis to determine the model nonlinearity for crack damage detection. Chondros et al. [15] analytically solved the governing equation of a cracked beam and determined the lowest frequencies of a simply supported beam with open or fatigue crack. An experiment was conducted to validate the frequency predictions. Above research studies have focused on the understanding of nonlinear behavior of a beam with a fatigue crack based on the bilinear model. Very few efforts focus on collecting insights from these nonlinear responses and developing effective schemes to conduct damage detection and quantify the crack depth.

In a recent paper, Andreaus et al. [16] used the two-dimensional finite element model along with a frictionless contact model to investigate a cantilevered beam with a breathing crack. Nonlinear dynamic responses including sub- and super-harmonics were characterized under a harmonic excitation. Peng et al. [17] introduced the concept of nonlinear output frequency response functions (NOFRFs) based on the *Volterra* series solution approach [18] to study the bilinear system, in which a polynomial function was used to curve-fit the bilinear spring restore force. The peaks in the NOFRFs can be used to detect the presence of a crack. Chatterjee [19] applied the same *Volterra* series approach to determine nonlinear response of a cantilevered beam with a breathing crack. A deterministic detection scheme was presented to investigate the first and second harmonic term in the response spectrum. Then, the damage assessment can be conducted to relate the crack severity to the ratio between these two harmonics through numerical simulations. These research works have built a solid foundation for understanding the nonlinear dynamics of a cracked beam and demonstrated the possibility of nonlinear vibration based damage detection.

Beck et al. proposed a Bayesian statistical framework to update a structural model and its associated uncertainties using dynamic response data as this framework increases accuracy along with a quantitative assessment of accuracy in response predictions [20]. Lam et al. applied the same Bayesian model updating method to characterize multiple cracks in a cantilever beam using transient vibration data [21]. The Euler-Bernoulli beam theory was adopted to determine analytical transient response, in which continuity conditions were applied at each crack location. A two-stage approach was used to determine the number of cracks, each crack depth and location. Experiments were conducted to verify this probabilistic crack detection approach. Other crack detection studies based on the same Bayesian probabilistic framework can be found in literature [22–24]. The probabilistic optimization approach is typically used to characterize the crack in these Bayesian inference efforts.

As aforementioned in the literature [21–24], the crack is assumed to be open at all times. Due to cycling loading, an actual crack experiences open and close in engineering structures. The objective of this paper is to extend the Bayesian approach to characterize the breathing crack condition in beam structures using nonlinear vibration responses. First, a single degree of freedom model is employed to model a beam with a single breathing crack. A polynomial function is used to represent the bilinear stiffness of the beam as used in Ref. [17]. The analytical approximate solution approach is applied to determine the nonlinear response based on the Modified Homotopy Perturbation Method (MHPM) [25,26]. Secondly, the Bayesian approach is employed for the parameter