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An algorithm for dynamic vehicle-track-structure interaction analysis for high-speed trains

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ABSTRACT

The objective of the present work is to develop a robust, yet simple-to-implement algorithm for dynamic vehicle-track-structure-interaction (VTSI) analysis, applicable to trains passing over bridges. The algorithm can be readily implemented in existing bridge analysis software with minimal code modifications. It is based on modeling the bridge and train separately, and coupling them together by means of kinematic constraints. The contact forces between the wheels and the track become Lagrange multipliers in this approach. A direct implementation of such an approach results in spurious oscillations in the contact forces. Two approaches are presented to mitigate these spurious oscillations – (a) a cubic B-spline interpolation of the kinematic constraints in time, and (b) an adaptation of an alternate time-integration scheme originally developed by Bathe. Solutions obtained using this algorithm are verified using a generic differential algebraic equation (DAE) solver. Due to high train speeds and possible track irregularities, wheels can momentarily lose contact with the track. This contact separation is formulated as a Linear Complementary Problem (LCP). With this formulation, including contact separation in the analysis amounts to replacing a call to a linear equation solver by a call to an LCP solver, a modification of only two steps of the procedure. The focus of this paper is on the computational procedure of VTSI analysis. The main contribution of this paper is recognizing computational issues associated with time-varying kinematic constraints, clearly identifying their cause and developing remedies.

1. Introduction

Vehicle-track-structure-interaction (VTSI) consists of the reciprocal influence of a bridge and a train on each other. As a train traverses a bridge, the deflections of the bridge as well as irregularities in the track act as support displacement input to the train at its wheels. The ensuing dynamics of the train in turn cause time-varying forces and vibration in the bridge. The purpose of VTSI analysis is to assure track safety and passenger comfort [1]. Passenger comfort is related to the acceleration experienced while the train is passing over the bridge [1]. Track safety depends on the rate of loading of the track with time-varying forces from the train. For high-speed trains, this rate of loading may coincide with natural frequencies of the bridge resulting in resonance and amplification of bridge response. Moreover, geometric track irregularities may cause magnification of the contact forces and damage to the wheels and the track [2], or even train derailment under extreme conditions, such as earthquakes [3] and collisions [4]. Therefore, assessment of vehicle running safety is another important reason for VTSI analysis.

Mathematically, VTSI can be represented as a system of two sets of equations of motion, for the train and bridge subsystems. Sun et al. [5] distinguish three types of algorithms for solving this system. The first group of algorithms aims to solve the system directly [2,6–8]. This approach is based on combining the two sets of equations into a single equation and solving the obtained equation. The second method requires condensation of vehicle degrees of freedom (DOF) into the bridge equation of motion and solving this updated equation. Based on such an approach, Yang et al. [9] proposed a vehicle-bridge interaction element. The third approach is to solve equations of motion of the vehicle and the bridge separately using iterative procedures [10–12]. As Sun et al. [5] have pointed out, the first type of algorithms cannot be easily implemented into existing structural analysis software due to the fact that vehicle and bridge models are combined together. The second method does not allow incorporating the various train models into analysis, and also requires specialized analysis software. The third approach is the most suitable in terms of incorporating a VTSI algorithm into existing software. However, as it was observed by Yang et al. [9], the VTSI problem involves a large number of contact

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points between the wheels and track, hence convergence of iterative procedures may be low. Sivaselvan et al. [13] proposed an algorithm that overcomes these drawbacks and can be integrated into existing software. The idea is to complement the system of equations of motion with a constraint equation and then solve the equations of motion of the vehicle and bridge separately using the constraint condition and avoiding an iterative procedure. In this case, a system of differential–algebraic equations (DAE) is obtained, which requires careful consideration of the numerical integration scheme. Moreover, adopting such a modular approach, contact separation between the wheels and the bridge can be easily modeled by formulating a linear complementary problem (LCP). The formulation of contact problems using complementarity methods has a long history [14], and, as a result, rolling wheel-rail contact can be modeled at various levels of detail [15–18]. Zhu et al. [19] proposed a similar approach applying the mode superposition method to the bridge.

In the present work, the VTSI algorithm proposed by Sivaselvan et al. [13] is expanded upon. The main goal is to develop a highly modular algorithm that can be incorporated into existing software without interfering with the bridge model formulation. However, the cost of this modularity is the necessity to solve a DAE system. As opposed to the approach proposed by Zhu et al. [19], the current algorithm employs a general finite element model of the bridge directly, without utilizing the mode superposition method, which allows broadening the usage of algorithm and applying it to different types of bridges.

The organization of the paper is as follows. In Section 2, a system of governing equations of motion is derived. Various time-integration approaches are discussed in Sections 3–5. Finally, in Section 6, contact separation between the wheel and the bridge is considered.

2. Governing equations

A conceptual model of a train passing over a bridge is illustrated in Fig. 1. The train is modeled as a sequence of cars. Each car is represented as a multibody system composed of rigid bodies, springs and dashpots. The bridge is modeled using standard structural or finite elements, such as beam and plate elements, box girders, cables, etc., i.e. any elements that can be utilized in commonly used bridge analysis software.

The following assumptions are used to develop the algorithm for two-dimensional VTSI analysis [13]:

1. The train moves on a straight-line path along the bridge (curved paths are a topic of current research and will be addressed in a subsequent paper), so that the train dynamics are entirely planar, and the train loads on the bridge are in the global Z direction.
2. The train and bridge displacements are small, so that linearized kinematics can be used for both. Material behavior is also linear.
3. The speed of the train is constant.
4. Through Section 5, the train wheels do not lose contact with the bridge (contact separation is considered in Section 6).
5. When a wheel is outside the span of the bridge, its displacement is zero.
6. At any time instant, each wheel is on only one element. If a wheel is on a joint, it is arbitrarily assigned to a bridge element connected to that joint (see Fig. 2).
7. Dead loads are applied to the bridge and the train using static analysis before dynamic analysis is performed.

2.1. Train model

The train is modeled as a sequence of cars, each of which is a multibody system. The equation of motion of the train can then be written as (1).

$$\mathbf{M}^{t}\ddot{\mathbf{u}}^{t} + \mathbf{C}^{t}\dot{\mathbf{u}}^{t} + \mathbf{K}^{t}\mathbf{u}^{t} + \left(\mathbf{L}^{t}\right)^{T}\boldsymbol{\lambda} = \mathbf{P}^{t} \tag{1}$$

where $\mathbf{M}^{t}, \mathbf{C}^{t}$ and $\mathbf{K}^{t}$ are the mass, damping and stiffness matrices of the train model, $\mathbf{P}^{t}$ is the external load vector, such as a constant self-weight load on the train model, and $\mathbf{u}^{t}$ is a vector of train displacements. Matrix $\mathbf{L}^{t}$ represents the influence of the reaction forces from the bridge on the train model. This matrix also plays a role in the constraint Eq. (6). Vector $\boldsymbol{\lambda}$ is the vector of contact forces between the train wheels and the bridge (positive downward on the bridge and upward on the wheels). Here superscript "t" stands for train.

2.1.1. Example

To illustrate how Eq. (1) comes about, a simple car model composed of a rigid bar connected to the wheels through dashpots and springs is considered. One such car is shown in Fig. 3a. The train is assumed to have N_{WH}^{t} wheels.

Using free body diagrams of the car and the wheels in Fig. 3b, the equations of motion of the train model are derived as (compare with Eq. (1)):

$$\begin{bmatrix} m^{c} & & & \\ & I^{c} & & \\ & & m^{w} & \\ & & & m^{w} \end{bmatrix} \begin{pmatrix} \ddot{u}_1^{t} \\ \ddot{u}_2^{t} \\ \ddot{u}_3^{t} \\ \ddot{u}_4^{t} \end{pmatrix} + \begin{bmatrix} 2c^{s} & 0 & -c^{s} & -c^{s} \\ 0 & \frac{c^{s}l^{c2}}{2} & \frac{c^{s}l^{c}}{2} & -\frac{c^{s}l^{c}}{2} \\ -c^{s} & \frac{c^{s}l^{c}}{2} & c^{s} & 0 \\ -c^{s} & -\frac{c^{s}c^{s}}{2} & 0 & c^{s} \end{bmatrix} \begin{pmatrix} \dot{u}_1^{t} \\ \dot{u}_2^{t} \\ \dot{u}_3^{t} \\ \dot{u}_4^{t} \end{pmatrix}$$
$$+ \begin{bmatrix} 2k^{s} & 0 & -k^{s} & -k^{s} \\ 0 & \frac{k^{s}l^{c2}}{2} & \frac{k^{s}l^{c}}{2} & -\frac{k^{s}l^{c}}{2} \\ -k^{s} & \frac{k^{s}l^{c}}{2} & k^{s} & 0 \\ -k^{s} & -\frac{k^{s}l^{c}}{2} & 0 & k^{s} \end{bmatrix} \begin{pmatrix} u_1^{t} \\ u_2^{t} \\ u_3^{t} \\ u_4^{t} \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -m^{c}g \\ 0 \\ -m^{w}g \\ -m^{w}g \end{pmatrix} \tag{2}$$

where m^{w} is the mass of each wheel; m^{c} and I^{c} are the mass and moment of inertia of the carriage; k^{s} and c^{s} are the stiffness and damping of each suspension, l^{c} is wheel-to-wheel distance; u_i^{t} are displacements of the train model.

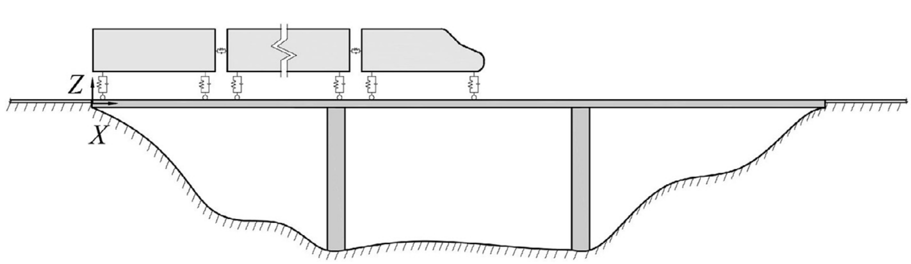

Fig. 1. Conceptual model of a train passing over a bridge.