Engineering Structures 143 (2017) 22–39

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A thermodynamics-based formulation for constitutive modelling using damage mechanics and plasticity theory

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ARTICLE INFO

Article history:
Received 24 November 2016
Revised 2 April 2017
Accepted 6 April 2017

Keywords:
Constitutive modelling
Thermodynamics
Damage mechanics
Plasticity theory
Coupled damage-plasticity
Concrete
Rocks
Ductile
Brittle

ABSTRACT

In this study, a generic formulation for constitutive modelling of engineering materials is developed, employing theories of plasticity and continuum damage mechanics. The development of the proposed formulation is carried out within the framework of thermodynamics with internal variables. In this regard, the complete constitutive relations are determined by explicitly defining a free energy potential and a dissipation potential. The focus is put on the rigour and consistency of the proposed formulation in accommodating the coupling between damage and plasticity, while keeping its structure sufficiently generic to be applicable to a wide range of engineering materials. In particular, by specifying the coupling between damage and plasticity in the dissipation function, a single generalised loading function that controls the simultaneous evolution of these dissipative mechanisms is obtained. The proposed formulation can be readily used for either enriching existing plasticity models with damage, or for the developments of new coupled damage-plasticity models. The promising features and the applications of the proposed formulation for describing the behaviour of different engineering materials are discussed in details.

1. Introduction

Computer simulations of the mechanical response of structures, by means of a numerical technique, such as finite element method (FEM), play a key role in many modern civil and mechanical engineering applications. The accuracy of analysis of any numerical simulation, however, depends on a constitutive model, capable of adequately capturing the material behaviour under complex loading scenarios. Theories of plasticity and continuum damage mechanics (CDM) have been widely used for the development of constitutive models in order to describe the inelastic behaviour of materials. At the macroscopic scale, inelastic behaviour can be observed as the reduction in strength and stiffness as well as the occurrence of residual strains. The observable macroscopic behaviour of materials is mainly governed by several underlying microscopic dissipative mechanisms. These dissipative mechanisms are the direct result of progressive, irreversible changes in the material microstructure. Examples of such changes are closure or expansion of micro-voids, micro-crack initiation and coalescence, frictional sliding between the two surfaces of microcracks, dislocation of defects in the crystal structure of metals and so forth. From a phenomenological perspective, the effects of all underlying mechanisms which cause the occurrence of residual deformations (e.g. frictional sliding, dislocation of defects, etc.) can be represented by a plastic strain tensor as a macroscopic variable. Similarly, the effects of all mechanisms giving rise to strength and stiffness degradation may be accounted for by a damage variable, which can be a scalar or a tensor of higher orders. In general, for any constitutive model, a set of internal variables is required for a complete description of inelastic behaviours of not only the current state but also the previous history of deformations [1–10].

During the course of inelastic deformation of engineering materials, plasticity and damage processes normally occur together and one influences the evolution of the other. Hence, constitutive models which take only one of these two mechanisms into account may not adequately represent the observed behaviour of materials. Formulations based merely on plasticity theory [11–19], for instance, generally suffer from limitations in capturing the stiffness reduction due to damage growth [11], although they may be successful in modelling the overall stress-strain response, by explicitly defining some kind of hardening/softening rules for the yield function. Elastic-damage models [20–27], on the other hand, can successfully capture the material stiffness reduction due to damage processes, yet they may be criticised for their inadequacy in properly modelling the residual strains due to plastic deforma-

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http://dx.doi.org/10.1016/j.engstruct.2017.04.018
0141-0296/

Nomenclature

Ψ	Helmholtz free energy potential
Φ	total dissipation rate function
Φ_v	dissipation rate function corresponding to volumetric plastic deformation
Φ_s	dissipation rate function corresponding to shear plastic deformation
Φ_D	dissipation rate function corresponding to damage
D	scalar damage variable
K	bulk modulus
G	shear modulus
ε_V	total volumetric strain
ε_S	total effective shear strain
α_V	volumetric plastic strain
α_S	effective shear plastic strain
ε_p	accumulative plastic strain
ε_{pc}	critical value of the accumulative plastic strain
σ_{ij}	stress tensor
S_{ij}	deviatoric stress tensor
J_2	second invariant of the deviatoric stress tensor
I_1	first invariant of the stress tensor
ε_{ij}	strain tensor
e_{ij}	deviatoric strain tensor
α_{ij}	plastic strain tensor
λ	non-negative multiplier
δ_{ij}	Kronecker delta
C_{ijkl}	elastic stiffness tensor
C^t_{ijkl}	tangent stiffness tensor
p	mean pressure
q	deviatoric stress
$\bar{\chi}_{ij}$	generalised stress tensor
$\bar{\chi}_V$	generalised mean pressure
$\bar{\chi}_s$	generalised shear stress
$\bar{\chi}_D$	conjugate damage energy
χ_{ij}	generalised dissipative stress tensor
χ_V	generalised dissipative mean pressure
χ_s	generalised dissipative shear stress
χ_D	conjugate dissipative damage energy
y	yield function in true stress space
y^*	yield function in generalised dissipative stress space
ϕ_v	function representing the effect of α_V in total dissipation
ϕ_v	function representing the effect of α_s in total dissipation
ϕ_D	function representing the effect of D in total dissipation
E	function of stresses and internal variables
F	function of stresses and internal variables
f_v	dimensionless function of stresses and internal variables
f_s	dimensionless function of stresses and internal variables
a	dimensionless function of stresses and internal variables
b	dimensionless function of stresses and internal variables
c	dimensionless function of stresses and internal variables
r_d	dimensionless function of stresses and internal variables
r_p	dimensionless function of stresses and internal variables
f_y	dimensionless function of stresses and internal variables
f_{cy}	dimensionless function of stresses and internal variables
f_{ty}	dimensionless function of stresses and internal variables
Q	ultimate stress (Von Mises model)
Q_t	ultimate stress in tension (parabolic Drucker-Prager model)
Q_c	ultimate stress in compression (parabolic Drucker-Prager model)
H	material parameter determining the rate of expansion of the yield surface
H_t	the value of parameter H in tension
H_c	the value of parameter H in compression
k	material shear strength (Von Mises model)
α	parabolic Drucker-Prager material parameter
β	parabolic Drucker-Prager material parameter
p_c	initial yield pressure under isotropic compression
p_t	initial yield under isotropic decompression (expansion)
ω	material parameter controlling the shape of the yield surface (geomaterials model)
γ	material parameter controlling the shape of the yield surface (geomaterials model)
ρ	back stress (geomaterials model)
M	slope of the final failure envelope (geomaterials model)

tions, which may only be included into these models by means of some empirical definitions [20]. Hence, a combination of both plasticity theory and CDM is necessary for the development of a realistic and rigorous constitutive model.

Significant efforts have been made during the past few decades to construct coupled damage-plasticity models by specifying the interaction between the two dissipative mechanisms. One of the existing approaches for coupling damage and plasticity is to employ two separate loading functions pertaining to damage and plasticity. In this approach, the two inelastic mechanisms are linked through the constitutive relations and the plastic yield function is expressed in the effective stress space, associated with the undamaged state of the material [8,28–51]. In these models, hardening rules are usually introduced to control the evolution of the yield function, while a softening rule controls the evolution of the damage function, and their coupling results in an overall hardening or softening behaviour, owing to the combined effects of both damage and plasticity. Nevertheless, due to the use of two separate loading functions, it is usually difficult to correlate these two surfaces with the experimentally obtained yield envelope and its evolution to failure, especially in multiaxial loading scenarios. In particular, the coupling between damage and plasticity can only take place if the inner loading surface (usually the plastic yield surface) evolves and hits the outer one, after which the two surfaces evolve together.

In another class of coupled damage-plasticity models [9,52–59], the above-mentioned issues associated with employing two loading surfaces are alleviated by explicitly defining the damage growth as a function of plastic strain. In these models, the only role of the damage function is to determine the onset of damage-induced softening, while the overall inelastic behaviour relies on the yield function and its flow rules. A physical interpretation of these models is that plasticity can be considered as an active mechanism of deformation and energy dissipation followed by damage as a passive mechanism, that is, damage can occur only after some plastic deformation has already taken place. Such