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Application of the smooth evolutionary structural optimization method combined with a multi-criteria decision procedure

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ABSTRACT

Topological Optimization (TO) of a structure with elastic-linear behavior considering a multi-objective problem is the goal of this work. For this process, an evolutionary heuristic formulation denominated SESO (Smoothing Evolutionary Structural Optimization) in conjunction with a finite element method was applied. In order to find the best topology, the authors choose to use the Analytic Hierarchy Process (AHP), which provides a simple but theoretically sound multiple-criteria methodology for evaluating alternatives. Together with the Weighted Sum Method (WSM), which is considering one of the best and simplest multi-criteria decision analyses or multi-criteria decision making method for evaluating a number of alternatives in terms of a number of decision criteria, this work aims to show the best topology among an interactive process focusing on four parameters: volume, displacement, stress and performance index. Some numerical examples are presented in order to show how the topological optimization process works and as such, to demonstrate the advantages of combining SESO, AHP and WSM as a structural optimization method.

1. Introduction

The optimization process aims to find a suitable model, according to one or more cost criteria, in order to minimize (or to maximize) design constraints. In other words, it can be stated that optimization is the search for a better model to meet a specific project requirement.

In the structural design field, there are three basic types of optimization. First, the parametric optimization, in which the structure presents a fixed shape and topology and only the constitutive materials used and/or the dimensions of the structural elements can change [1]. Furthermore, the shape optimization, in which the structure presents a fixed topology, varying its shape and not having cavity insertions in the domain of the model [2]. Finally, the topological optimization, in which is a generalization of other optimization types, allowing both the insertion of cavities in the initial domain and the remodeling of the initial boundary shape [3]. Moreover, there is the topography optimization, although is a very special type of shape optimization that aims to find a distribution pattern of the reinforcement of structures, such as plate and shell [4].

Sometimes, it is necessary to systematically and simultaneously optimize a collection of objective functions, and this process is called multi-objective optimization (MOO) or vector optimization [5,6]. Multiple conflicting criteria could need to be handled, and satisfying a group of criteria may not be so easy. Multi-objective optimization thus deals with such conflicting objectives, providing a mathematical framework towards an optimal design state which accommodates the various criteria demanded by the application.

In order to solve a problem with multiple objectives, there is a tool that helps decision making called Analytic Hierarchy Process (AHP). According to [7], it is one of the most widely used multiple-criteria decision-making tools. This method consists in comparing pairs of attributes measured on a scale of priorities. This scale is defined by the user, following any kind of criteria, in which an absolute score has been given to each attribute, and a higher value represents that one element dominates another as regards that pair of attribute. At the end of the process, the corresponding weights are obtained and applied to a process called WSM, used extensively not only to provide multiple solution points by varying the weights consistently, but also to provide a single solution point that reflects preferences presumably incorporated in the selection of a single set of weights [8].

Evolutionary methods need to perform an iterative process to achieve the optimal topology, generating partial responses. To determine the end of the optimization process, these methods

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require a stopping criterion, whose commonly adopted parameter is the volume. Often however, the topology obtained at the end of the iterative process whose final volume is equal to the volume adopted as stopping criterion, it does not actually correspond to the optimal desired topology. Therefore, we use the AHP to find the weights and use them in the WSM to determine the optimal topology from the weighing concerning the four parameters evaluated in the structure: volume, displacements, stress and performance index.

Evolutionary methods can be used to find solutions for different problems. First, [9] apply the ESO topological method to obtain the strut-and-tie model for reinforced concrete structures, considering the nonlinear material behavior of concrete. Otherwise, [10] present a topological optimization procedure applied to the design of a support plate for an eolic turbine. Also, [11] mention that robust topological optimization minimizes the sensitivity to structural uncertainties and imperfections, which are conditions not considered in the initial design but very important for the structural performance and safety. Finally, [12] design a bicycle structure using the topological optimization.

The use of new isotropic and non-homogeneous materials, such as Functionally Graded Materials (FGM) are growing in the last years. The optimization procedure presented in this paper can be used if the kinematics is included in the finite elements. [13] present analytical solutions for the bending and free vibration analysis suitable for simply supported plates. Bending, shear deformation and thickness stretching effects are considered for the FGM analysis. [14] study the shear deformation and thickness stretching effects together with thermal effects in the FGM analysis. More recently, [15] present a new hyperbolic shear deformation theory applicable to bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates.

Also, [16] presents a deterministic algorithm used to obtain the optimal topology that penalizes the density of the finite elements. This kind of algorithm is computationally expensive and uses an approximated stop criteria based on the variation of the objective function value. According to [17], the objective function is not normally available for most of the topological optimization problems found in real engineering applications. Moreover, according to [18] the calculation of the gradient can be computationally expensive and for those cases where the objective function presents discontinuity or functions with some degree of complexity it becomes more complicated to obtain a response. Optimization algorithms based on volume removal criteria, such as those presented in [10] and [19], not guarantee that the final reached volume is the real volume corresponded to the optimum topology. The present work aims to improve the search for the optimum topology, avoiding the calculation of the gradient necessary to obtain the hessian matrix.

2. Smoothing evolutionary structural optimization (SESO)

SESO is a variant of the ESO (Evolutionary Structural Optimization) method, developed by [8], consisting of a gradual removal of finite elements previously generated, in regions that do not effectively contribute to the structure ("inefficient materials") by any of the criterion for rejection. It is an heuristic topology optimization method that gradually and smoothly removes finite elements as compared with the classical ESO. This procedure starts with a discretization of the entire structure in a fixed finite element mesh, also named design or extended domain, which includes the boundary conditions (forces, displacements, cavities and other initial conditions) of the elastic problem to be solved iteratively via FEM. Afterwards, the von Mises stress is evaluated at each element, and the highest stress value of the whole structure, the maximum von Mises stress, is taken as a reference to be used in inequality (1a), as described in [19].

$$\sigma_e^{vm} < RR_k \cdot \sigma_{i,max}^{vm} \tag{1a}$$

$$RR_{k+1} = RR_k + RE \tag{1b}$$

where σ_e^{vm} is the equivalent von Mises stress of the i-th element in the iteration and $\sigma_{i,max}^{vm}$ is the maximum effective stress of the structure in the iteration "i"; RR_k is the Rejection Ratio at the k-th steady state ($0.0 \leq RR_k \leq 1.0$), which is an input parameter that is updated using the Rate of Evolution (RE). The evolutionary process is defined by adding the rate of evolution to the Rejection Rate, presented in Eq. (1b), which is applied to control the removal of the elements during the evolutionary process.

Hence, the elements that satisfy Eq. (1a) are gradually removed from the mesh by an iterative process until a steady state is achieved. This removal is conducted by altering the constitutive matrix, for example, by setting a very small value to that element stiffness, which makes this element inefficient without the need to remake the mesh in the problem.

However, just removing the elements in the iteration according to Eq. (1a), can often lead to an early or precipitated withdrawal of elements that should not be removed. This happens often seeing that during the evolutionary process, a certain element that should not have been taken out was removed just to reach this equation. The generated solution is forced, non-optimal and possibly generates an unstable region called "chessboard" (or "checkerboard"), considered to be one of the major problems in the topological optimization of structures that occurs due to the poor conditioning model of the stiffness matrix, which means inadequate convergence analysis [20,21]. To solve this problem, SESO proposes an organization of elements that does not satisfy Eq. (1a) such that (p%) of these elements are removed and (1 − p%) is returned to the structure (Fig. 1). This return is accomplished by a regulating function that performs a smoothing procedure or, in other words, it weights elements with a higher potential for removal and removed elements that can be returned to the mesh. This proce-

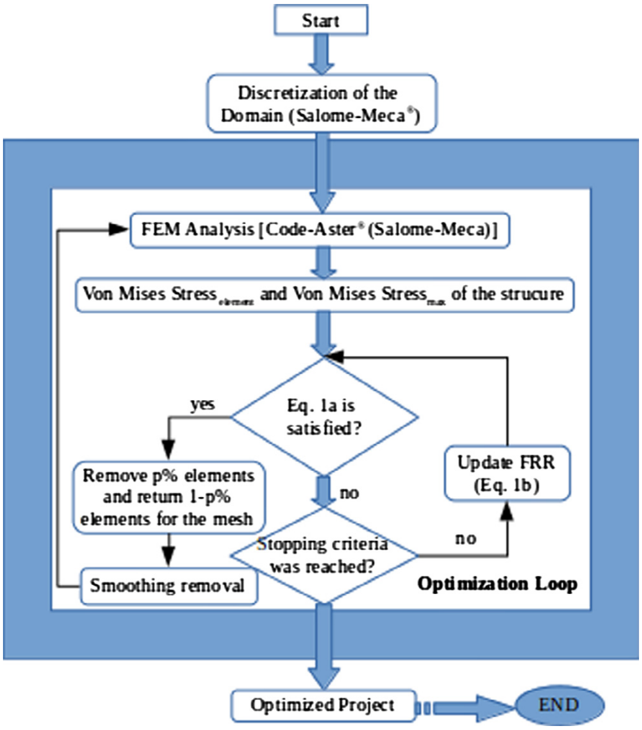

Fig. 1. Flowchart of the SESO method including the software Salome-Meca®.