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Evaluation of the lateral stability of precast beams on an elastic bearing support with a consideration of the initial sweep

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ABSTRACT

This study assessed the lateral behavior and stability of precast concrete beams under elastomeric bearing pad conditions. The lateral instability failure of the beam occurs due to the lateral displacement induced by the self-weight and rotation along the beam combined with a rigid body rotation at the support. Therefore, accounting for all the components of the lateral displacement, an analytical equation that can calculate the critical weight of a beam related to the lateral instability of the beam was derived. Furthermore, this study evaluated the critical lateral movements, including the displacement and rotation, with a variation in the initial sweep in the beam. Finally, an analytical procedure was also developed to evaluate the safety of the precast concrete beam against the lateral instability of the beam with a consideration of the initial sweep. The critical values obtained from the proposed method would be used to establish control values for retaining the lateral and torsional stability of a precast concrete beam on the elastic bearing support.

1. Introduction

Precast concrete beams are widely used in bridge design and construction, and the use of precast beams to improve structural performance and to facilitate rapid construction is increasing. In addition, the development of high-strength materials and prestress technology has allowed precast concrete beams to become longer and more slender. However, the design and construction of longer and more slender beams has created concern regarding lateral instability failure, which has not been considered a primary failure mode in concrete beams [1]. In particular, the lateral instability of precast concrete beams due to insufficient lateral support should be taken into greater consideration. The instability failure of precast concrete beams during construction causes economic losses associated with construction delays, possible damage and harm to construction equipment and workers, as well as the potential closure of roadways [2]. In recent years, the collapse of several long precast concrete beams, such as the collapse of I-girders in Pennsylvania and Arizona in the US [3], has generated considerable apprehension about critical lateral behavior related to lateral instability of beams, which invariably includes initial geometric imperfections.

As an initial study, Laszlo and Imper [4] proposed an analytical procedure to calculate the rollover safety of a precast concrete beam during handling and transportation. Based on the work of Laszlo and Imper [4], Mast [5,6] provided a method to calculate the degree of safety against the rollover failure of a precast concrete beam on flexible supports. The method proposed by Mast [5,6] has been accepted by the PCI Bridge Design Manual [7], and is used to evaluate the lateral stability of precast concrete beams during handling and transportation. For a precast beam rotated about the center of the flexible support, the rotational angle combined with the flexural displacement caused by the weight of the beam and the initial imperfect displacement in the lateral direction leads to an overturning moment, M_t, which is resisted by the rotational stiffness of the bearing support. Thus, with a small rotational angle of the beam at the support, θ_s, the equilibrium equation can be expressed as:

$$(wL)(z_o\theta_s + e_i + y_{cg}\theta_s) = k_r(\theta_s - \alpha) \quad (1)$$

where w is the weight of the beam per unit length, L is the length of the beam, z_o is the average of the lateral flexural deflection due to the weight of the beam, e_i is the initial lateral eccentricity with respect to the roll axis, and y_{cg} is the height of the center of gravity of the beam above the roll axis, k_r is the rotational stiffness of the support, and α is the initial rotational angle of the support. Mast [6] then defined the factor of safety (FS) against the overturning of the beam as the ratio of the resisting moment to the overturning moment as follows:

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Notation

A_b	area of bearing support
b_w	width of web of beam
C_r	coefficient of reduction due to initial sweep in beam
C_w	warping constant
E	modulus of elasticity of beam
E_s	compressive modulus of elasticity of bearing support
EC_w	warping rigidity of beam
EI_y	flexural rigidity of beam
e_i	initial lateral eccentricity of beam with respect to roll axis
G	shear modulus of beam
GJ	torsional rigidity of beam
h	height of cross-section of beam
h_b	total elastomer thickness of bearing support
I_b	moment of inertia of bearing support about longitudinal axis of beam
I_y	moment of inertia of beam in weak axis
J	torsional constant of beam
k_r	rotational stiffness of bearing support
k_v	vertical stiffness of bearing support
L	length of beam
M_t	overturning moment of beam
M_{cr}	elastic lateral and torsional critical moment
r	radius of stability
u	lateral displacement of beam
u_m	lateral displacement at mid-span of beam
u_m^{cr}	critical lateral displacement at mid-span of beam
v_m	vertical displacement at mid-span of beam
w_{sw}	self-weight of beam per unit length
w	weight of beam per unit length
w_{cr}^o	critical weight of perfect beam per unit length
w_{cr}	critical weight of beam with initial sweep per unit length
x_i	values obtained from nonlinear numerical analysis
$\bar{x}_i$	values calculated using the proposed equations
y_{cg}	height of center of gravity of beam
z_o	average lateral flexural deflection of beam
α	initial rotational angle of support
$\delta(x)$	initial sweep at x of beam
δ_o	initial sweep at mid-span of beam
$\theta_t(x)$	torsional angle at x of beam
θ_t^{mid}	torsional angle at mid-span of beam
θ_t^{avg}	average torsional angle along length of beam
θ_m	total rotational angle at mid-span of beam
θ_m^{cr}	critical total rotational angle at mid-span of beam
θ_s	rotational angle at support of beam
θ_s^{cr}	critical rotational angle at support of beam

$$FS = \frac{r(\theta_s^{cr} - \alpha)}{z_o\theta_s^{cr} + e_i + y_{cg}\theta_s^{cr}} \tag{2}$$

where r is the radius of stability, which is calculated from the rotational stiffness of the support, k_r, divided by the total weight of the beam. Eqs. (1) and (2), however, require the critical rotational angle of the support at failure, θ_s^{cr}, to calculate the critical weight of the beam, which induces lateral instability of the beam, and an *FS* against the lateral instability failure of the beam. The PCI Bridge Design Manual [7] recommends that only for the beam during transportation, the critical rotational angle, θ_s^{cr}, (called as the maximum rotational angle in the PCI Bridge Design Manual) can be assumed to be the height of the roll center above the road and the transverse distance from the centerline of the beam to the center of the truck wheel. However, for the precast beam on an elastic support, no analytical method or procedure has been proposed to determine the critical rotational angle, θ_s^{cr}, which induces lateral instability of the beam.

Burgoyne and Stratford [8] excluded the initial imperfection when deriving an equilibrium equation, so the proposed equation can be solved to obtain the critical weight of a beam on an elastic support related to the lateral instability of the beam. For a small rotational angle of the beam at the support, θ_s, the equilibrium equation derived by Burgoyne and Stratford [8] can be expressed as:

$$(wL)[y_{cg}\theta_s + z_o\theta_s] = 2k_r\theta_s \tag{3}$$

where the left-side term is the overturning moment due to the rotation of the beam at the support and the flexural displacement along the beam in the minor axis, and the right-side term is the resisting moment at the elastic bearing support. Eq. (3) is basically based on a geometrically perfect beam assuming that the beam rotates as a rigid body at the support with no variation in twist along the length of the beam and no resistance to rotation about the vertical axis.

For the beam with an initial lateral displacement (sweep), Burgoyne and Stratford [8,9] recommended that given the critical weight and initial sweep of the beam, the Southwell [10] plot can be used to obtain the lateral deflection of the beam, u, due to the variation in the weight of the beam, w:

$$u = \frac{\delta_0}{1 - \frac{w}{w_{cr}}} \tag{4}$$

where w_{cr} is the critical weight of the beam, and δ_o is the initial sweep in the beam. On the other hand, Allen and Bulson [11] indicated that the square of the ratio of the real weight to the critical weight of the beam in the Southwell plot is more appropriate for the lateral and torsional behavior of a beam according to the following equation:

$$u = \frac{\delta_0}{1 - \left(\frac{w}{w_{cr}}\right)^2} \tag{5}$$

Mandal and Calladine [12] demonstrated that the standard Southwell plot provides satisfactory estimates even for the lateral and torsional buckling problems. The modified form of the Southwell plot using the square of the load exhibited a deviation from the linear trend lines for low-level magnitudes of load. In addition, the modified Southwell plot was valid only in the case where only one kind of imperfection, either an initial sweep or twist, exists.

Furthermore, Vidigal de Lima and El Debs [13] performed numerical analysis to discuss the effect of nonlinear behavior of concrete material on the lateral stability of slender rectangular concrete beams. The result of the study showed that concrete did not exhibit any cracks until a rotational angle of 15°, which means that the lateral instability failure of a concrete beam, particularly slender and longer beams, occurs before the development of cracks in the concrete. In addition, Vidigal de Lima and El Debs [13] indicated that the lateral instability of a concrete beam is strongly affected by the lateral bending stiffness, but not by the torsional stiffness. That is, there is little variation in twist along the beam when lateral instability occurs.

Recently, Lee [14,15] investigated the lateral behavior and instability associated with the initial imperfection and thermal deformation for precast I-girders on an elastic bearing support.