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A second-order flexibility-based beam-column element with member imperfection

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ABSTRACT

In this paper, a new flexibility-based beam-column element with member initial imperfection is proposed for direct analysis or second-order inelastic analysis of steel structures. In past decades, the stiffness-based elements gain great successes in handling large deflection problems while the flexibility-based elements play a dominant role in plastic analysis. Apart from the merits of the above two types of elements, the proposed flexibility-based type element considers member initial imperfection and distributed plasticity along the member length and therefore one element is generally sufficient to model a member for practical design. On the contrary, several stiffness-based elements are required to model a member in order to capture the plastic behavior of member under complex loads. The conventional flexibility-based type elements cannot meet the codified requirements of direct analysis as they do not take member initial imperfection into account and this limitation is removed in the new element proposed in this paper. Several examples are given in this paper to illustrate the accuracy and validity of the proposed element.

1. Introduction

The terms “Direct Analysis”, “Second-order Analysis” and “Advanced Analysis” appear in many modern design codes such as AISC-LRFD [1] and Eurocode-3 [2]. All these analysis methods essentially aim to assess the member and system stability in an integrated manner in place of separated individual member check adopted in the traditional linear analysis with the effective length method (ELM). The motivation of the new design method is that the analysis outcome can be directly used for design intent without further uncertain capacity reduction for buckling check using the effective length factor, provided that the analysis method can reliably capture the structural behavior with consideration of important factors affecting member and system strength. If one or more key factors are excluded, the design method will be classified as “Indirect Analysis”.

It is well known that the second-order P-Δ and P-δ effects, initial geometrical imperfections in both member and system levels, residual stress, material yielding and joint behavior should be included in the direct analysis as these factors may significantly contribute to the deflection and instability affecting the computed stress. The most difficult but vital consideration is to incorporate the member initial bowing to an analysis process so that the stability of member and system as well as their interaction can be taken into account. The structural model using one element per member in direct analysis is highly preferred for efficient and accurate modeling of initial out-of-straightness, which is unavoidable for real structural member and required for consideration in design codes. In contrast, modeling of a member by several elements is undesirable since more extensive modeling effort and longer computer time will be required. Generally speaking, the directions of the member initial imperfections following the lowest global buckling mode are considered most favorable in a few design codes.

Chan and Zhou [3] and Liu et al. [4,5] proposed several stiffness-based (also well known as displacement-based) beam-column elements allowing for member initial imperfection for second-order direct analysis (SODA). Chan and Gu [6] developed an initially curved stability function which can accurately trace the nonlinear behavior of slender members even under extreme cases when buckling dominates. In conjunction with refined plastic hinge technique [7], the above works are extended to inelastic analysis of steel and composite structures [8,9]. As the element stiffness matrices of the works [3–6] are based on explicit integration without need of using the numerical quadrature rule such as Gauss quadrature for capturing of plasticity effects, excellent numerical efficiency and fast convergence are reported. Ziemian and McGuire

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[10] proposed a modified tangent modulus approach considering residual stress and plastic hinge for second-order inelastic analysis. Their method is simple and can be easily incorporated into conventional stiffness-based beam-column elements, but the geometrical imperfections are not included at element stiffness. Also, their model requires calibration for selected cross-section subjected to major or minor axis bending.

Extensive works have been done on flexibility-based (also known as force-based) beam-column elements [11–15]. Neuenhofer and Filippou [12,13] made significant contribution on the practical use of flexibility-based element with consideration of material yielding and geometrical nonlinearity. The authors [13] also proposed a curvature-based displacement interpolation (CBDI) method to determine the transverse displacement of the element using the kinematic relations and the curvatures at integration points. To formulate element tangent stiffness and resisting forces, an element state determination approach based on the Newton-Raphson method was introduced in Ref. [13]. Nukala and White [16] presented four state determination algorithms for geometric and material nonlinear beam-column elements which follow the two-field Hellinger-Reissner (HR) variational principle. De Souza [14] developed beam-column elements for planar and spatial frame analysis based on Hellinger-Reissner (HR) functional to derive the weak form of equilibrium and compatibility equations. With discretization of the frame section into a number of fibers as well as insertion of numerical integration points along the member, the flexibility-based elements are widely used for distributed plastic analysis. It is noted that there is no flexibility-based element proposed until recently for explicit consideration of member geometrical imperfections and therefore, these previous elements cannot meet the requirement of design codes such as Eurocode-3 [2] and CoPHK [17].

Fairly speaking, the stiffness-based elements show unique superiority for geometric nonlinearity due to computational efficiency and high accuracy while the flexibility-based elements are suitable for material nonlinearity because of strict satisfaction of force equilibrium at the element level. The stiffness matrix of stiffness-based elements (e.g. Chan and Zhou [3] and Chan and Gu [6]) can be directly obtained through exact integration while the flexibility-based elements generally need to adopt numerical integration (e.g. Neuenhofer and Filippou [13]) to form the flexibility matrix with complicated procedure for element state determination. Thus, it is clear that the flexibility-based element requires more computational effort in forming stiffness matrix and computing unbalanced forces during the incremental-iterative nonlinear solution.

The stiffness-based elements equipped with refined plastic hinge technique [18] can generally handle the concentrated plasticity at member ends or an additional plastic hinge along the member length [4,5], and therefore they may give results of inadequate accuracy for more complicated plastic behavior of a member under complex loading scenarios, for instance, several point loads applied at the different locations of a member. Although multi elements per member can alleviate this problem, the approach brings inconvenience in modeling the member initial imperfections and increase computer time and modeling efforts.

From the above, a flexibility-based beam-column element considering member imperfection (abbreviated to FBMI hereafter) based on Hellinger-Reissner (HR) functional is useful for practical uses of the element and proposed in this paper for second-order direct analysis (SODA). This new element is extended from Neuenhofer and Filippou [13] by incorporating member initial bowing at the element level for three-dimensional frame analysis. Fiber section approach is adopted to account for the distributed plasticity of a member. It is noted that many members may remain elastic for practical structures under various design load cases as not all members could be loaded to plastic range in a particular load case attaining limiting deflection, structural integrity and architectural requirements. Thus, it is recommended that the stiffness-based elements and the new flexibility-based element can be used in an integrated manner under the SODA design, in which the former is adopted for modeling of the members under low stress level, and the latter is used for the members under high stress. This integrated treatment will significantly enhance computational efficiency and therefore a more rational SODA method can be developed for routine design. Several examples are employed to validate the accuracy and efficiency of the proposed element along this line of thought.

2. Element formulations

The proposed flexibility beam-column element (FBMI) is extended from Neuenhofer and Filippou [13] by incorporating member initial bowing at element level for three-dimensional frame analysis. The warping and shear deformation are neglected and the applied loads are assumed conservative, i.e. the loads are increased proportionally and independent of the load path, and nodal loads are assumed with distributed load or concentrated load lumped to element end nodes.

The derivation of the proposed element will be summarized in this section. To simplify the element formulations and improve numerical efficiency, the new element is developed in the basic coordinate system with six natural degrees of freedom. The kinematic hypothesis for the element is presented. The Hellinger-Reissner (HR) functional is introduced to generate the equilibrium and compatibility equations. The section constitutive relation and the consistent flexibility matrix are described. The curvature-based displacement interpolation (CBDI) technique [13] is adopted to determine the displacements which are not explicitly assumed as opposed to the conventional displacement-based elements. Finally, the co-rotational approach is used to transform the stiffness matrix defined in the basic coordinate system to the global coordinate system so that the proposed element can be incorporated into the conventional finite element package directly.

2.1. Basic coordinate system

The proposed beam-column element with six element degrees of freedom is formulated in the basic coordinate system with the rigid body modes suppressed. Further, the stiffness matrix and associated kinematic variables are transformed to the global coordinate system such that each element node has six degrees of freedom (DOF), i.e. three translational and three rotational DOFs, as seen in Fig. 1.

The element end forces **P** and the corresponding end displacements **D** defined in basic system are given below,

$$\mathbf{P} = \{P_1P_2P_3P_4P_5P_6\}^T = \{NM_{Iz}M_{Jz}M_{Iy}M_{Jy}T\}^T \tag{1}$$

$$\mathbf{D} = \{D_1D_2D_3D_4D_5D_6\}^T = \{u\theta_{Iz}\theta_{Jz}\theta_{Iy}\theta_{Jy}\psi\}^T \tag{2}$$

2.2. Kinematic hypothesis

On the basis of Bernoulli-Euler beam theory and ignoring warping effects, the displacement field of the space beam-column element can be expressed as,

$$\mathbf{u}(x,y,z) = \begin{Bmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - y \cdot v'(x) - z \cdot w'(x) \\ v(x) - z \cdot \psi(x) \\ w(x) + y \cdot \psi(x) \end{Bmatrix} \tag{3}$$