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# Beam element for thin-walled beams with torsion, distortion, and shear lag

Francisco Cambronero-Barrientos [a,*], Julián Díaz-del-Valle [b], José-Antonio Martínez-Martínez [a]

[a] Department of Civil Engineering, University of Burgos, C/ Villadiego s/n, 09001 Burgos, Spain
[b] Department of Structural and Mechanical Engineering, University of Cantabria, Avda. de las Castros s/n, 39005 Santander, Spain

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ABSTRACT

Practical design of bridges and other structures requires the use of quick and simple calculation methods, rather than the use of tridimensional models using shell or solid finite elements. These methods have to be used for a general loading state, taking into account the different structural mechanisms, and generating the results required to apply the verifications of the structural codes and to understand the structural behavior. In this work, a beam-type element is proposed to adress these objetives for the case of thin walled sections. This element has three nodes, with five-degrees of freedom per node, more than the six degrees of a conventional 3D beam, incorporates the effects of shear lag, torsion and distortion homogeneous and non homogeneous in the distribution of normal stress. Various examples were tested to verify the validity of the beam element according different calculation methods.



## 1. Introduction

In the design of a structure its structural behavior may be calculated for a solid sections in an acceptable way using the classic Euler–Bernoulli beam theory and the Saint–Venant torsion theory. However, the theory of V.Z. Vlasov [26] has to be used for a thin-walled section, to take into account the case of cross-sections that will not remain flat and to incorporate to the calculation the torsional warping, the sectional deformation, and the influence of shear strain in the distribution of normal stress.

The finite-element method with shell or solid elements is widely used for stress/strain analysis in structures with thin-walled sections, such as the deck of a bridge with a thin-walled girder. The finite element method takes into account the overall strength-related aspects of the cross-section, but doesn't provide information on the descomposition of the results as that are provided as an addition of the effects. An estimation of that descomposition is desirable for the design of the structure: to predict its operation, and by doing so optimizing its design; and, to apply structural codes and standards in which the different verifications are required to be carried out separately in relation to different resistance modes, and not with total stresses results. In addition, the implementation of shell or solid tridimensional models in the finite-elements method requires to consider in a big number of freedom degrees what implies to increase importantly the calculation an postprocessing time.

Over time, beam-type elements have been developed to include additional attributes to the conventional 3D beam. In 1983, some researchers among whom S.H. Zhang, L.F. Boswell and L.P.R. Lyons [4,28,29] developed a beam-type element for the analysis of structures with thin-walled sections. The curved element is defined by three nodes, each with nine degrees of freedom, and incorporates torsional and distortional warping. Shear lag is calculated with effective width coefficients.

In 1987, Li Guohao [7] developed a curved element with all resistance modes, but not incorporating the shear-lag effect for normal stresses.

In 1990, Ferdinando Laudiero and Marco Savoia published a work [14–16] about the influence of tangential strain in bending and torsion in beams with thin-walled sections. Analysis of the warping function is proposed by means of the integration of the tangential shear stress. Throughout the beam, the longitudinal displacement is defined by the product of the warping function, which only depends on the coordinates of the cross-section, by an intensity function that depends on the longitudinal coordinate of the

* Corresponding author.
*E-mail addresses:* fcambronero@ubu.es, estructuras@vettones.com (F. Cambronero-Barrientos), julian.diaz@unican.es (J. Díaz-del-Valle), jamartinez@ubu.es (J.-A. Martínez-Martínez).

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beam. On the basis of the displacements field and minimizing the total potential energy, a system of differential equations are defined to solve various particular cases.

In 1991, A. Ghani Razaqpur and Hangang G. Li [22,24] developed a two-node beam element with a straight directrix, for beams with thin-walled sections, which included all the resistance modes. Subsequently, in 1994, they incorporates to multi-cell curved beams [23]. The element incorporates torsional and distortional warping, and additional degrees of freedom were added to consider shear lag, that represents the warp of the upper and lower parts of the cross-section, using quadratic equation functions (see Fig. 16c). The additional degrees of freedom depends on the cross-section shape, so it is not an element for general use, but restricted to a serie of cross-sections shapes.

In 1993, A. Prokić [17–21] presented a beam-type element especially designed to consider the effect of shear lag under normal stresses. The element has a straight directrix and three nodes. The cross-section is considered perfectly rigid on its plane, so the distortion resistance mode was not included. The cross-section is discretized with a variable number of nodes, assigning a degree of freedom for each one of them at the extreme nodes of the beam, which represents the warp and implies that the cross-section no longer remains flat. It may be said that rather than a beam-type element, in which there is only discretization in the longitudinal direction, it is a finite tridimensional element in which there are both longitudinal and transversal discretization within the cross-section.

Between 2012 and 2013 M.J. Andreassen and Jeppe Jönsson using GBT (generalised beam theory) instead of finite elements, develop a distortional semi-discretized thin-walled beam element, in which the cross-section is discretized into wall elements and the analytical solutions of the related GBT equations are used as displacement functions in the axial direction. [1–3,11].

Wang XiaoFeng et al. in 2014 published a beam element with general thin-walled closed cross-section [27]. He includes non–homogeneous torsion, but does not include distortion and shear lag.

In 2016 Dongil Shin et al. [25] present a finite element beam for analysis of tapered thin-walled box beams. The element is for a particular case and not for any shape of cross-section.

As it has been explained, various researchers have developed one-dimensional finite elements for resistance mode of combined calculation on thin-walled beams. Each one of them lack some characteristic able to make them become generally applicable. The general shortage is the non-inclusion of shear lag with no restrictions on the section shape.

Hence, in the present work, the development of one-dimensional finite element is proposed, with the minimum possible number of degrees of freedom, which includes all the structural mechanisms, and that is valid for any thin-walled section without restrictions. The starting point in the development of this type of finite element is the selection of a general calculation method of the cross-section structural mechanisms.

Steen Krenk and Bo Jeppesen published a method by means of the finite elements to find the stress distributions, warping functions, and cross sectional properties of thin-walled beam section, in 1989 [13], for uniform shear and Saint–Venant torsion. The cross-section is discretized in two-node elements, with four degrees of freedom in total, one per node and two internal each of them, that provide the exact solution to the problem. Compared with calculation methods based on analytical equations for the behavior of the cross-section, the method they presented is easily implemented in a computer program for any section shape, whether to open or closed sections. This method is a development of an earlier one by S. Krenk and O. Gunneskov [12] in which the cross-section is discretized in a bidimensional model with triangular finite elements to obtain its properties.

The Danish researcher, Jeppe Jönsson published an enhacement of the work by Steen Krenk and Bo Jeppesen [8], in 1998 [13], in which the finite-elements they proposed extended to the formulation of the mechanical behavior of warping torsion and distortion of thin-walled sections. The finite element they used had two nodes, each of them with a degree of freedom that represented the warp of the node, using linear functions to interpolate within the finite element. The author applies resultant cross-sectional properties in the analytical solution of distortion [9,10].

In this article, a one-dimensional element that was first developed in 2013 [6] is presented. On the basics of the cross-sectional properties calculation method, the element incorporates all the structural mechanisms without restriction.

## 2. Formulation of the thin-walled beam element

The goal of this section is to obtain the stiffness matrix of the beam element. Afterwards main notation and sign conventions are presented, the deformation modes are described, and also a summary of how to evaluate them, based on Jönsson [8], whose paper is essential to understand correctly this paper.

Then the displacement field is composed by the summatory of the displacement field of all deformation modes. Shear forces $Q_x$ and $Q_y$ appear on the displacement field as unknows, so it is neccesary to modify it to make them disappear. That will be possible by the orthogonalization of the warping shear functions $w_x$ and $w_y$. After this modification, total potential energy will be calculated and the stiffness matrix of the element is obtained.

### 2.1. Notation and sign conventions

The following notation will be used. The cross-section is contained within the $x$ and $y$ coordinate directions, with the beam axis progressing along the $z$ coordinate. The displacements $\{v_x, v_y, v_z\}$ are positive when they occur in a positive direction along the axes, and the rotations $\{\theta_x, \theta_y, \theta_z\}$, when they occur in the positive direction according the right hand rule. This convention is represented in Fig. 1.

The displacements of any point in the space are denoted as $\{v_x, v_y, v_z\}$ and for that reason those displacements are functions of $(x, y, z)$. And $\{u_x, u_y, u_z\}$ are the displacements of any point of the beam axis and they are functions of $(z)$.

The shear stresses, $\tau_{zs}$, are considered positive when their direction is the same as the forward direction of the wall under consideration, and the normal stresses are considered positive when their direction is the same as the forward direction of the $z$ axis. This notation is shown in Fig. 2b. $(t_x, t_y)$ is the unit tangent vector of a wall. The sign convention for internal forces is shown in Fig. 2a.

### 2.2. Basic assumptions and considerations

The basic hypotheses considered on the behavior of the cross-section are as follows. Each wall of the cross-section is in plane

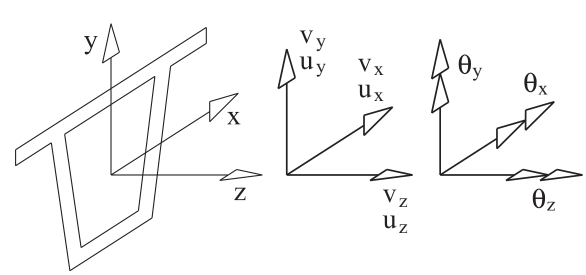


**Fig. 1.** Notation for displacements and rotations.