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Energy based load-impulse diagrams

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ABSTRACT

A load-impulse (P-I) diagram is a useful tool in preliminary strength design and damage assessment. Current approaches to derive P-I diagrams are primarily based on numerical dynamic analyses of the structural system. Further, there are no general energy based solutions that define P-I relationships, and very little has been done to address energy flow as a basis for P-I diagrams. Since the behavior of a structural member is governed by the amount and rate of external energy input and its ability to dissipate energy, defining energy flow relationships will characterize the entire domain of structural responses and corresponding damage. This paper presents a comprehensive energy flow approach to enable one to define the entire P-I domain, and illustrates the application of an energy-based P-I diagram. The results are validated using test data and numerical tools, such as DSAS.

1. Introduction

A response spectrum is a graphic representation of a peak value of a structural response parameter to a specific excitation, as a function of frequency [1]. A load-impulse (P-I) diagram is obtained by performing a transformation of coordinates on a structural response spectrum [2–4], and it enables an expedient assessment of structural damage for a given load. As shown in Fig. 1, with a defined response limit, the diagram indicates the load-impulse combinations that can determine the level of damage and if the component will fail. As noted in those references, both the response spectrum and the P-I diagram have asymptotes that can be computed based on energy principles.

Although a P-I diagram is a useful tool in preliminary strength design and damage assessment, there are no general energy based solutions that define these relationships. Available energy considerations have the following limitations:

- The Energy Balance method is only applicable to the impulsive and quasi-static asymptotes. The response in the dynamic domain (or regime) is more complicated and is significantly influenced by the profile of the load history. Therefore, a comprehensive energy based solution would consider energy transitions and (loading) rate effects.
- Current energy considerations for P-I diagrams address only the final state of a structural element (peak response). However, the entire loading history and associated dynamic response are important for characterizing the behavior.
- Although energy based approaches have been widely applied for solving engineering problems, very little has been done to address energy flow as a basis for P-I diagrams.

An energy-based approach for load-impulse diagrams for structural members under a wide range of dynamic loads will represent a fundamental improvement in the field of protective structures.

2. Methodology

The correlation between load-impulse diagrams and energy flow must be determined in order to obtain energy based solutions for P-I diagrams. Therefore, a rational analysis is needed to describe the energy flow history of dynamic structural responses for given load pulses.

Generally, for dynamic analyses, the mechanical characteristics of a structural system can be adequately replicated by properly assuming a simplified dynamic model. In addition, a reliable material and constitutive model can suitably describe the relationship between the dynamic resistance and the response of the structure. For example, a numerical approach to analyze the dynamic response of reinforced concrete members subjected to concentrated transient loads based on a single-degree-of-freedom (SDOF) formulation and reliable material and constitutive models was pre-

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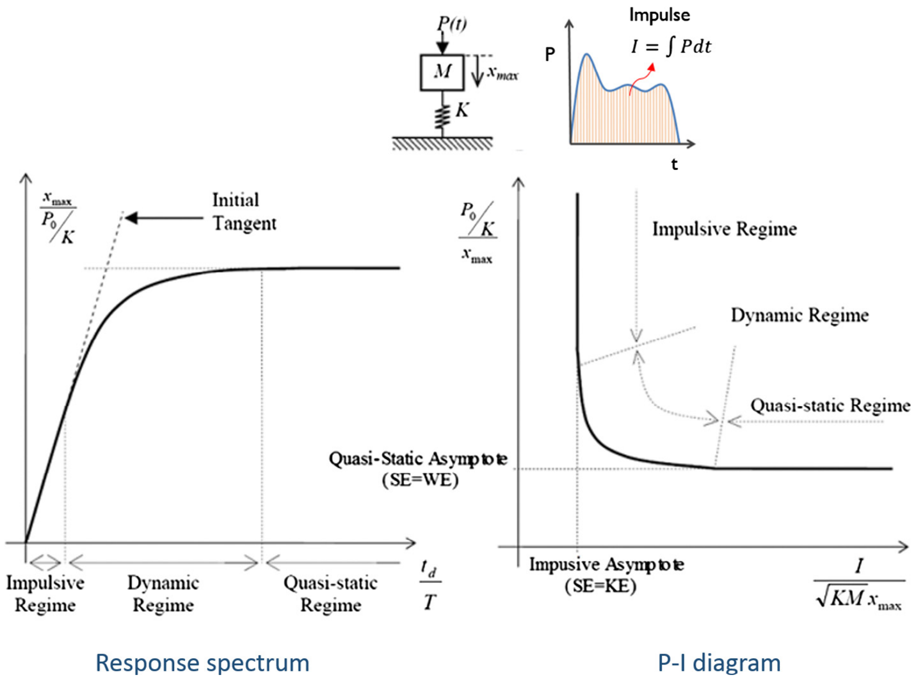

Fig. 1. Typical response spectrum and P-I diagram [3].

sented in [5]. Thus, this study employs the same approach for dynamic analyses.

The effect of loading rate on a reinforced concrete element is influenced by the loading rate sensitivity of the mechanical characteristics of the materials involved. The behavior of a structural element under impact loading may consist of two response phases: the local response due to the stress wave that occurs at the loading area during a very short period after impact; and the global (or overall) response after impact, including the free vibration effect that occurs, over a long period throughout the entire structural member after impact. Thus, it is of fundamental importance to understand the effect of loading rate on structural elements and the relationship between energy flow and load-impulse diagrams.

2.1. Characteristic of energy flow

For structural dynamic analysis, the structural response under impact is strongly related to the load duration and the structure's natural period [2,6]. Generally, the relationship is separated into three domains: impulsive, dynamic, and quasi-static. Each domain represents different relationships between the load function and the structural response.

A dynamic analysis for a flexural SDOF system subjected to a given rectangular load pulses with different load durations illustrates how energy flow transforms from one type of energy to another in the corresponding domain (Fig. 2). The energy expressions for the SDOF system under a given load pulse are functions of peak load, mass, stiffness, natural frequency, time, and load duration.

In the impulsive regime, the load duration is very short compared to the response time of the system, which is affected by the system's natural period. The load is applied and removed before the structure experiences a measurable deformation. The strain energy (*SE*) curve barely rises before the end of the load duration, t_d. The maximum response (at time t_m) can then be assumed to be independent of the load time history (or load profile). For the case of the dynamic regime, which is between the impulsive and quasi-static regimes, the loading duration and the system response time are of the same order. The response in this regime is more complicated because it is influenced by the profile of the load function; i.e. the structural response cannot be decoupled from the load. For the quasi-static regime, the load duration is significantly longer than the response time. The load barely dissipates before the structure achieves the maximum deformation, and the response in this regime depends only upon the peak load, the load pulse shape, and the structural resistance.

It can be observed from the energy transitions for each regime that the sum of kinetic (*KE*) and strain (*SE*) energies equal the work done by the external load (*WE*) at all times, and they are complementary and time dependent.

Based on the results from the dynamic analyses and the influence of the load pulse on P-I diagrams, the amount of energy imparted by the load pulse depends not only on the peak load but also on the load duration (and the load pulse shape), which implies that the amount of energy input and its rate are important for describing the structural response.

2.2. Energy based solutions for P-I diagrams

The energy balance method is the original approach for P-I diagrams derivation proposed by Baker et al. [2]. It is convenient to apply because two distinct energy formulations always exist, which separate the impulsive loading domain from the quasi-static loading domain, and greatly reduces computation efforts. However, its formulation is only applicable to the impulsive and quasi-static domains of the response spectrum. The dynamic domain of the load-impulse curve must be approximated utilizing suitable analytical formula. Baker recommended the following hyperbolic tangent squared relationship to approximate the transition region for linearly elastic oscillators [2]:

$$SE = WEtanh^2\sqrt{\frac{KE}{WE}} \tag{1}$$

Instead of applying the approximation formula, a general energy-based solution is presented here to define the entire load-impulse domain, based on the Law of Conservation of Mechanical