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Lateral-torsional buckling of simply supported anisotropic steel-FRP rectangular beams under pure bending condition

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ABSTRACT

In this paper, a generalized analytical approach for lateral-torsional buckling of simply supported anisotropic hybrid (steel-FRP), thin-walled, rectangular cross-section beams under pure bending condition was developed using the classical laminated plate theory as a basis for the constitutive equations. Buckling of such type of hybrid members has not been addressed in the literature. The hybrid beam, in this study, consists of a number of layers of anisotropic fiber reinforced polymer (FRP) and a layer of isotropic steel sheet. The isotropic steel sheet is used in two configurations, (i) in the mid-depth of the beam sandwiched between the different FRP layers and (ii) on the side face of the beam. A closed form buckling expression is derived in terms of the lateral, torsional and coupling stiffness coefficients of the overall composite. These coefficients are obtained through dimensional reduction by static condensation of the 6×6 constitutive matrix mapped into a 2×2 coupled weak axis bending-twisting relationship. The stability of the beam under different geometric and material parameters, like length/height ratio, ply orientation, and layer thickness, was investigated. The analytical formula is verified against finite element buckling solutions using ABAQUS for different lamination orientations showing excellent accuracy.

1. Introduction

A thin-walled slender beam subjected to bending moments about the strong axis may buckle by a combined lateral bending and twisting of the cross-section. This phenomenon is known as lateral- torsional buckling. Theory of thin-walled open section beams including axial constrains for isotropic materials was developed by Vlassov [1]. This classical theory neglects the shear deformation in the middle surface of the wall so that for the composite beams, the shear deformations may significantly increase the displacements and reduce the buckling loads. The shear deformation theory for transversely loaded isotropic beams was developed by Timoshenko and Gere [2].

Many researchers then started to study the lateral torsional buckling for the laminated composite beams using different theoretical approaches and enhancing their work with experimental programs and finite element models to validate the theory. Lin et al. [3] studied the stability of thin-walled composite member using the finite element method. Seven degrees of freedom at each node for each two-nodded element were used to model the fiber reinforced plastic. The seven degrees of freedom are the dependent translations in three perpendicular directions and the corresponding rotations in addition to the angle of warping. The stiffness matrices of a beam element were used to develop the element shape functions. A number of examples of thin walled-open sections were solved, different cross sections like channels, I sections, and Z-sections were tested as well as different boundary conditions. The study concluded the importance of the influence of in-plane shear strain on the critical buckling load for lateral torsional buckling and combined torsional and flexural modes. It also minimized the significance of shear strain effect on critical buckling when the buckling happens in terms of a flexural mode. Davalos and Qiao [4] used the non-linear elastic theory to develop a stability solution for lateral-distortional buckling for composite wide flange beams based on the principle of total potential energy. A fifth-order polynomial shape function was adopted for the displacement field construction. Then, the proposed model was validated against two geometrically identical experimental beams loaded at mid-span, with different material characteristics. A good agreement was obtained against the experimental results and a finite element model. Kollár [5] presented a stability analysis of thin walled composite columns under axial loading conditions. A closed form solution was derived using a modified version of Vlasov's classical theory (1961) for isotropic material to account for the composite action. The effect of shear deformation in in-plane

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displacements and in the restrained warping was examined and a shear matrix was formulated in addition to the bending matrix. Lee et al. [6] studied the lateral buckling of composite laminated beams. An analytical approach based on the classical lamination theory was derived for different boundary conditions and different laminate stacking sequences. The examined beams were tested under various loading configurations and various locations. The beams were then compared against a one dimensional finite element model under different load configurations. The model showed a good agreement against the finite element model of simply supported I beam in cases of pure bending, uniformly distributed loads, and central point load. Yet, the model was not appropriate for pure bending with off-axis fiber orientation due to coupling stiffness. Sapkás and Kollár [7] offered closed form solutions for simply supported and cantilever, thin walled, open section, orthotropic composite beams subjected to concentrated end moments, concentrated forces, or uniformly distributed load. The solution indirectly accounted for shear deformation by adjusting the bending and warping stiffness of the composite beams. Qiao et al. [8] formulated an analytical solution for flexural-torsional buckling of composite cantilever I beams based on an energy method developed from the non-linear plate theory. A good agreement against finite element method was obtained. Furthermore, four different cantilever beams were tested experimentally under tip loads to examine the flexural-torsional response. Also, good agreements were shown against the experimental results. Kotelko [9] presented a theoretical analysis of local buckling which represents material failure. This study covered different cross sections of thin walled beams and columns. These cross sections varied between lipped and plain channels as well as box-section. This theory matched previous theories in a way that it depends on the rigid-plastic model. Yet, it mainly differs by considering a constitutive strain-hardening of the used material. This analytical approach is particularly useful in the initial phase of design process and may be applied as a simplified design tool at the early stage of design process, including crush-oriented design. Karaagac et al. [10] tested the stability of a cantilever laminated composite beam under static and dynamic conditions. A linear translation spring was attached to the beam to control the lateral deformation. The attached elastic support location varied between the free end and the mid-span of the beam. Length-to-thickness ratio, variation of cross-section in one direction, orientation angle, static and dynamic load parameters, stiffness and position of the elastic support were the main variables to study the stability of the beam. Numerical polynomial approximations for the displacements and the angle of twist were derived and showed a reasonable accuracy against the finite element method. Machado [11] derived an analytical solution for lateral stability of cross-ply laminated thin-walled simply supported bisymmetric beams subjected to combined axial and bending loads. The presented theory included shear deformability and took into account large displacements and rotations; moderate bending rotations and large twisting angles. The proposed solution also examined the nonlinear pre-buckling geometrical deformation for more accurate representation of the lateral stability conditions. The buckling loads obtained analytically were, in general, in good agreement with the bifurcation loads observed in the post buckling response. The study concluded that the buckling moments computed from classical theory is overestimated. Also, it presented pre-buckling and post buckling displacement curves to relate the stiffness behavior of the beam to the applied loads and also to study the fiber orientation against the buckling loads.

In this study, an analytical model applicable to the lateral-torsional buckling of simply supported anisotropic hybrid (steel-FRP), thin-walled, rectangular cross-section beams, subjected to pure bending is developed. This model is based on the classical plate lamination theory (CPT), and accounts for the arbitrary laminate stacking sequence configurations. The analyzed beams consist of six layers of fiber reinforced polymer (FRP) sheets and one isotropic steel sheet even though the solution is applicable to any number of layers. The FRP sheets have the same thickness and the same mechanical properties, yet they vary in terms of fiber angle orientation. The location of the steel sheet was examined in order to understand its influence on the lateral torsional buckling critical moment. A sandwich stacking configuration (ST-I) is defined by placing the steel sheet in the mid-thickness of the beam. A sided stacking configuration (ST-II) is defined by placing the steel sheet at the side face of the beam. A series of FRP angle configurations were determined for comparisons against a finite element model and also to compare the different configurations against each other. The finite element model is developed in ABAQUS to predict critical buckling moments and compare with the results obtained from the analytical model. Also, the length of the beam to its height ratio and FRP layer thickness were examined to study the effect of beam size and thickness on the lateral torsional buckling resistance.

2. Analytical formulation

A simply supported hybrid (steel-FRP) laminated composite beam with length L and a thin rectangular cross section is subjected to pure bending at the ends, as shown in Fig. 1. The beam tends to buckle under a lateral-torsional behavior because of its small thickness.

The model in this study is based on the classical plate lamination theory, Kollár and Springer [12] and Barbero [13], which is derived from plane stress state, and all the assumptions in classical plate theory remain valid for laminated composite thin-walled beams.

2.1. Kinematics

Based on the assumptions in the classical plate theory, the displacement components u, v, w representing the deformation of a point on the plate profile section are given with respect to mid-surface displacements u_0, v_0, and w_0 as follows:

$$u(x,y,z) = u_0(x,y) - z\frac{dw_0}{dx}(x,y) \tag{1}$$

$$v(x,y,z) = v_0(x,y) - z\beta(x,y) \tag{2}$$

$$w(x,y,z) = w_0(x,y) \tag{3}$$

where $\beta = \frac{\partial w_0}{\partial y}$

The strains associated with small displacements from the theory of elasticity are given by

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x \tag{4}$$

$$\varepsilon_y = \varepsilon_y^0 + z\kappa_y \tag{5}$$

$$\gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} \tag{6}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \varepsilon_y^0 = \frac{\partial v_0}{\partial y}, \gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \tag{7}$$

$$\kappa_x = -\frac{\partial^2 w_0}{\partial x^2}, \kappa_y = -\frac{\partial \beta}{\partial y}, and \kappa_{xy} = -2\frac{\partial \beta}{\partial x} \tag{8}$$