Engineering Structures 146 (2017) 140–147

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Lateral-torsional dynamic instability of uniform and double-tapered rectangular beams under harmonic shear deformation

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ARTICLE INFO

Article history:
Received 7 March 2017
Revised 23 April 2017
Accepted 22 May 2017

Keywords:
Double-tapered beams
Harmonic excitation
Lateral-torsional instability
Parametric resonance
In-plane shear deformation

ABSTRACT

This paper analyzes lateral-torsional dynamic instability of elastic beams with rectangular cross section having constant thickness, with the depth symmetric with respect to the midpoint and either uniform or tapered linearly in each half. Free vibrations are also investigated. The ends of the beam are prevented from twisting and are pinned with respect to bending in the weak direction. In the strong direction, one end of the beam is fixed and the other is subjected to transverse harmonic motion. This problem was motivated by "butterfly-shaped links" proposed for use in seismic mitigation. Uniform torsion is assumed. Frequencies of free vibration are computed, critical excitation frequencies for lateral-torsional instability are determined, and critical combinations of excitation amplitude and frequency are obtained. The effects of geometric parameters of double-tapered beams on instability are presented. Lateral-torsional instability may occur for very small amplitudes of the moving end of the beam, as is typical for such problems involving parametric resonance.

1. Introduction

An elastic beam with rectangular cross section, having its depth linearly tapered in both halves symmetrically, is depicted in Figs. 1 and 2. A quasi-static analysis of this type of problem was investigated for the case $a \leq b$ (butterfly-shaped beam) in [1]. The term "double-tapered" here refers to the case of a cross section with constant thickness and linearly varying depth in each half of the beam (rather than a beam with varying thickness and depth), including both $a < b$ (as in Fig. 1) and $a > b$, along with $a = b$ as a special case.

As described in [1], butterfly-shaped beams have potential application in structural fuses to mitigate seismic response. The quasi-static analysis of [1] is extended here to harmonically varying shear deformation, as sketched in Fig. 3. It will be shown that large lateral-torsional motions may occur for small excitation amplitudes. Dynamic response and instability depend on the natural frequencies of the beam. Therefore free vibrations will be analyzed first.

In-plane free vibrations for double-tapered beams with rectangular cross section and fixed ends were studied in [2–5]. Results for lateral free vibrations for the same type of beams but with pinned ends with respect to the weak direction can be obtained from [6,7]. With respect to torsional vibrations, [8,9] examined I-beams with double-tapered webs and pinned ends.

Lateral-torsional dynamic instability under harmonic excitation involves parametric resonance [10–13]. In the equations of motion, the excitation is a factor in the coupling terms involving both torsion and weak-direction bending. The situation is similar to that for Mathieu and Hill equations, although here the harmonic excitation factors are multiplied by expressions that depend on the excitation frequency. Also, the diagonal terms in the parametric (stability) matrix are zero in the present work.

For small excitation amplitudes, it will be shown that the most important resonances (which have wedge-shaped instability regions in the undamped case) for lateral-torsional dynamic instability are combination resonances involving the sum of a natural frequency for weak-direction bending and a natural frequency for torsion.

Some problems involving lateral-torsional dynamic instability of uniform beams with rectangular cross section have been presented in [14–18], with the excitation in the strong direction. In [14,15], a horizontal cantilever was subjected to harmonic vertical deflection at the fixed end. Analyses and experiments were described, focusing on combination resonance. Instability of a cantilever with harmonic forces and moments at its tip was studied in [16]. In [17,18], the response of a vertical cantilever moved harmonically at its upper fixed end and containing a mass at its lower end was investigated.

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http://dx.doi.org/10.1016/j.engstruct.2017.05.043
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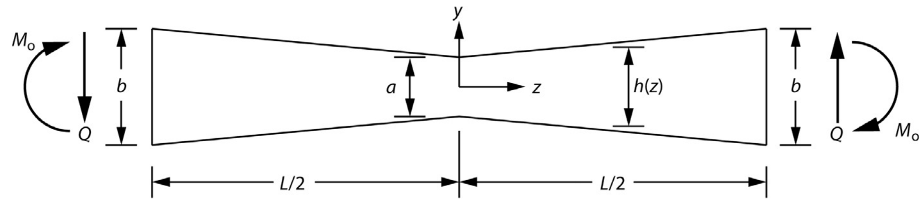

Fig. 1. Side view of beam.

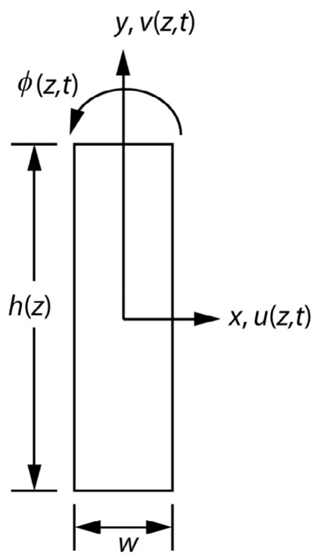

Fig. 2. Cross section of beam.

Uniform I-beams subjected to unequal dynamic moments at their ends were analyzed in [19,20]. The ends prevented twist and were pinned with respect to weak-axis bending. The moments contained static and harmonic components. Due to the static component, instability regions associated with a single natural frequency may be wedge-shaped, and combination resonances were not considered.

As far as the author can ascertain, the problem considered here (see Figs. 1–3), for uniform beams as well as for nonuniform beams, has not been analyzed previously. The governing equations of motion for double-tapered beams will be formulated in Section 2. In Section 3, free vibrations will be examined. The special case of a uniform cross section will be treated in Section 4, followed by double-tapered beams in Section 5. Concluding remarks will be given in Section 6.

2. Formulation

Consider the beam shown in Figs. 1 and 2. Positive senses of quantities are shown in these figures. The axial coordinate is z, with $z = 0$ at the midpoint of the beam. The coordinate in the weak direction is x and the coordinate in the strong (in-plane) direction is y, with corresponding deflections $u(z,t)$ and $v(z,t)$, respectively, where t is time. The angle of twist is $\phi(z,t)$.

The length of the beam is L. The ends of the beam are assumed to prevent twist and to be pinned with respect to weak-axis bending $u(z,t)$. In the strong direction with motion $v(z,t)$, the left end is fixed and the right end also has no slope and is moved up and down harmonically with amplitude Δ and frequency ω (Fig. 3).

The depth of the beam is $h(z)$, with $h(0) = a$ (at the waist) and $h(L/2) = h(-L/2) = b$ (at the ends). For the right half of the beam, $h(z)$ is given by

$$h(z) = a + 2(b-a)\frac{z}{L}, \quad 0 \leqslant z \leqslant \frac{L}{2} \tag{1}$$

The ratio a/b will be called the taper ratio and will sometimes be denoted λ. The cross section is rectangular with thickness w and area $A(z) = h(z)w$.

The density of the beam is ρ, the modulus of elasticity is E, Poisson's ratio is ν, the shear modulus is G, the torsional constant is $J(z)$, the moments of inertia for bending in the strong and weak directions are $I_x(z)$ and $I_y(z)$, respectively, and the polar moment of inertia is $I_p(z)$. Formulas for I_x, I_y, I_p, and G are given by

$$I_x(z) = \frac{1}{12}wh^3(z), I_y(z) = \frac{1}{12}w^3h(z), I_p(z) = I_x(z) + I_y(z),$$
$$G = \frac{E}{2(1+\nu)} \tag{2}$$

The major-axis bending stiffness is EI_x, the minor-axis bending stiffness is EI_y, and the torsional rigidity is GJ.

Due to the rectangular cross section, the loading, and the boundary conditions, uniform (St. Venant; pure) torsion is assumed to be appropriate. For a cross section with depth h and thickness w, and defining $r = h/w$, the value of the torsional constant J is taken to be [1]

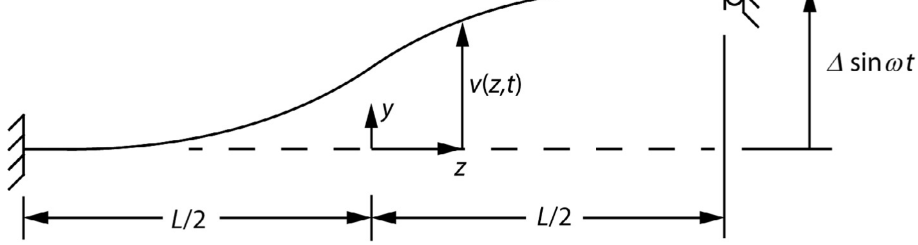

Fig. 3. Sketch of in-plane deflection v.