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A theoretical model for a piezoelectric energy harvester with a tapered shape

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ABSTRACT

A piezoelectric energy harvester made of a tapered cantilever surface bonded with piezoelectric patches is developed to harness energy from ambient vibrations. Compared with the available cantilever harvester of a uniform shape in the length direction, this harvester has a higher energy harvesting efficiency since a maximum collected power at each piezoelectric patch on the cantilever can be achieved. The current available models for cantilever harvesters are not applicable for the new developed tapered harvester due to the difficulties in dealing with the tapered shape. A corresponding finite differential model is hence developed to model the tapered harvester for estimating its efficiency by examining a governing differential equation with variable coefficients. The influences of some practical considerations, such as the geometry of the tapered cantilever and the width of piezoelectric patches on the root mean square of the generated electric power, are discussed. The results from the developed model show that an electric power up to 70 times higher than the available uniform cantilever harvesters can be achieved by the tapered harvester. This research provides an effective model for evaluating the high efficiency of the piezoelectric coupled tapered cantilevers in energy harvesting.

1. Introduction

Piezoelectric energy harvesting devices converting ambient energy into electrical energy have attracted much interest in both the academia and industry. Their applications include wearable electronics, where energy harvesting devices can power or recharge cellphones, mobile computers, radio communication equipment, high power output devices (or arrays of such devices) deployed at remote locations to serve as reliable power stations for large systems. The most common type of piezoelectric devices is the piezoelectric/metal sandwich beam mounted as a cantilever due to three major considerations. First, large mechanical strains can be produced directly owing to the piezoelectric effect of the materials during vibrations. Second, the construction of piezoelectric cantilevers is simpler compared to other harvesters. Third, the resonance frequency of the fundamental flexural modes of a cantilever is much lower than the other vibration modes of the piezoelectric element. So far, studies on piezoelectric energy harvesting technologies reported to date have involves a unimorph or bimorph cantilever design [1–5].

The exact expressions for the voltage, current, power, and tip deflection of a Timoshenko piezoelectric cantilever were derived [6]. Subsequently, several case studies were presented to examine the frequency response of vibration-based energy harvesters using this model. An analytical approach based on the Euler–Bernoulli beam theory and Timoshenko beam equations for the voltage and power generation was provided and compared with the electrical equivalent circuit and energy method [7]. The technique to adjust the performance of a piezoelectric bimorph vibrating in the flexural mode through axial preloads was studied to effectively scavenge energy from ambient mechanical vibrations/noise with varying-frequency spectra [8]. Based on a design of a bimorph cantilever located at an arbitrary position on a simply supported slender bridge, a piezoelectric power generation from moving loads was formulated [9]. An optimal design of a piezoelectric coupled cantilever structure attached by a proof mass subjected to seismic motion was introduced to achieve a higher efficient energy harvesting from high-rise buildings [10]. Two types of sea wave piezoelectric cantilever energy harvesters with proof masses were introduced and optimized to harvest electric energy from longitudinal or transverse wave motions of water particles [11,12]. Their results show that the harvesters can generate a power up to 55 W and 30 W for a practical longitudinal and transverse wave motions, respectively. A novel piezoelectric energy harvester

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device driven by a cantilever system to harvest energy from high-rise buildings was demonstrated [13,14]. This harvester can provide an efficient and practical energy harvesting in the process of dissipating the vibration energy of high-rise buildings. Dhakar et al. [15] proposed a 21-mm-long polymer extension beam with a proof mass of 0.72 g at its free end. The other end of the beam is clamped firmly to the free end of a 32-mm-long PZT-5A bimorph cantilever. Results showed that this design can tune the natural frequencies of the combinational cantilever notably. Azizi et al. [16] investigated the mechanical behavior of a bimorph piezoelectric micro cantilever exposed to harmonic base excitation, and the effect of load resistance and the effective piezoelectric stress constant on the equivalent damping ratio of output circuits including parallel and series connections. The effect of the impact of water droplets releasing from different various heights on the piezoelectric energy harvesting was studied [17] and the results show that a power output for one unit at around 2.5 μW. An auxiliary cantilever with a piezoelectric patch glued at its fixed end subjected to a resonance was conducted to improve power output [18]. Another innovative S-shaped meandering cantilever was designed to achieve a low resonance frequency below 30 Hz by reducing the stiffness of the beam [19]. A two-stage design was proposed in which a spring-mass system was designed to respond at the input frequency of the host structure, and then excite an array of piezoelectric cantilevers to harvest the mechanical energy [20]. A theoretical study of a similar two-stage piezoelectric energy harvester for human walking was conducted. In the study, ferromagnetic structures were used to tune the natural frequency of a piezoelectric cantilever with a magnetic proof mass [21]. Liu etc. [22] presented a power generator array based on thick-film piezoelectric cantilevers with different resonance frequencies to improve frequency flexibility and power output. The cantilever array can be tuned to the frequency and expanded the excited frequency bandwidth in ambient low frequency vibration. Xu et al. [23] presented a low frequency piezoelectric energy harvester "CANDLE" consisting of cantilever beam and cymbal transducers based on piezoelectric single crystal. The design uses a cantilever as the driving mechanism for two cymbal transducers to generate electrical energy.

Three major principles used in the available piezoelectric cantilever harvesters mentioned above to improve their power output are: (1) decreasing the natural frequency of the cantilever device by using a proof mass attached to its free end or a combination beam; (2) enhancing the excitation frequency to the vicinity of the natural frequency of the cantilever by a tuned resonance device; and (3) broadening the band response with respect to the excitation from the host structures. In the aforementioned energy harvester designs, the common intention is to make the piezoelectric coupled cantilever working at the resonance to achieve a maximum power output. However, many ambient vibration sources possess a spectrum of random frequencies. In these situations, tuning a piezoelectric energy harvester to a specific resonance frequency may not be an effective approach to improve efficiency because even a small amount of fluctuation in the input frequency will result in a large drop in the power output. Meanwhile, the tuned frequency device and proof mass may also increase the space and weight of the energy harvester device, and hence decrease the efficiency of the harvesters. And more importantly, the above-mentioned harvester devices only fully use the maximum strain at the fixed end which leaves the piezoelectric material under-utilized, resulting in sub-optimal efficiency [24].

The electrical energy from a piezoelectric material is proportional to the induced mechanical strain. Hence it is desirable to maximize the strain at each point in a beam to fully utilize the potential energy in piezoelectric materials. Under an optimal condition, the strain distribution in the beam would be completely uniform within the strain limit. The consideration suggests a triangular or trapezoidal shape cantilever which has a linearly increased wider cross-section, while reducing the size and weight of a piezoelectric cantilever. Baker et al. [25] introduced a piezoelectric harvester prototype made of a trapezoidal geometry cantilever by finite element method to examine the effect of geometry of the cantilevered piezoelectric beam on the power density. The results showed that the output power is 50% higher than that of a comparable rectangular cantilever with an even strain distribution throughout the beam. Glynne-Jones et al. [26] built a prototype of a thick-film PZT coupling trapezoidal cantilever, and then studied the effect of the load resistance and beam amplitude on the power output of the harvester. Mateu and Moll [27] provided a comprehensive study on a triangular piezoelectric bending beam structure for energy harvesting using shoe inserts. The results showed that by using a triangular cantilever, the strain along axial direction is made to be a constant and the harvested energy increases with an increase in the thickness of the triangular cantilever. Roundy [28] provided a relationship between the relative bending energies and strain profiles for alternative cantilever geometries, and found that a trapezoidal geometry can supply more than twice of the energy (per unit volume PZT) than the rectangular geometry. Reducing the coupled governing differential equations with variable coefficients to a pair of uncoupled second-order differential equations, Yuan et al. [29] simplified the governing equations for the free vibration of circular Timoshenko beams with both geometrical nonuniformity and material inhomogeneity along the beam axis and derived a series of exact analytical solutions from the reduced equations for the first time.

These researches aforementioned on cantilever energy harvester with varying cross-sectional area use either simulation or model testing, and focus on the cantilever tapering in the width because there is no analytical solution to the dynamic governing equations of the piezoelectric coupled cantilever tapering in both width and thickness directions. In fact, a taper in thickness of a cantilever has a greater contribution on the surface strain of the cantilever than that from the taper in width.

It is hence expected to develop a mathematical model for a tapered cantilever energy harvester in both width and thickness directions to harvest energy from ambient vibration. With this novel model, the effect of the taper in width and/or thickness of the cantilever, the width of the pzt patches, the width and the thickness at the fixed end of the cantilever on the energy harvesting efficiency of the tapered harvester can be studied comprehensively. The research findings are significant and helpful in designing the most efficient and economic piezoelectric coupled tapered cantilever energy harvesters which can maximize the surface strain at each point in the beam to harvest the strain energy at a fullest extent.

2. Introduction of the analytical model

The introduction of a piezoelectric coupled cantilever harvester with a rectangular cross section tapered in both thickness and width is depicted in Fig. 1(a and b). The cantilever is subjected to a sinusoidal force $F = Y\sin\omega' t$ at its free end. Fig. 1a schematically illustrates the normal section view of the harvester with a length of l. it is tapered in the thickness direction from h_0 at the fixed end to h_1 at the free end, and attached by pzt4 patches with the same thickness of t_p one by one on the upper and lower surfaces of the cantilever. Fig. 1b is the top view of the harvester tapered in the width direction from b_0 at the fixed end to b_1 at the free end. The attached individual pzt4 patches fully on the surfaces of the cantilever are with a varying length $l_p(x)$. Based on the model